

Phillips curve flattening mechanically weakens NAIRU identification

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Abstract

We show that, in the standard random-walk state-space model, the likelihood information about an interior NAIRU state, relative to the smoothness precision supplied by the state equation, equals $\gamma_2^2 R/2Q$. It is therefore proportional to the squared Phillips curve slope. Canadian quarterly data from 1977 to 2026 show the mechanism is quantitatively severe: across the February 1991 inflation-targeting break this ratio falls by a factor of twenty-three, driven almost entirely by the slope. The credible interval for the unemployment gap then contains zero in 97 per cent of quarters, against 73 per cent before, and without a sign restriction the posterior probability of a negative slope falls from 0.99 to 0.78.

Keywords: NAIRU; Phillips curve; state-space models; Bayesian estimation

JEL: E24; E31; E52; C11

1 Introduction

The NAIRU is an unobserved state, recovered from the comovement of inflation with the unemployment gap. One literature estimates it in state-space form and reports wide uncertainty bands ([Kuttner, 1994](#); [Staiger et al., 1997](#)); another documents the flattening of the Phillips curve ([McLeay and Tenreyro, 2020](#); [Hazell et al., 2022](#)). This note shows the two are mechanically linked.

We derive, for a general time-varying Phillips curve loading, the posterior precision matrix of the NAIRU path. Its diagonal gives a local statistic, $\Lambda_t = \gamma_{2,t}^2 R/2Q$: the likelihood information supplied by observation t relative to the smoothness precision supplied by the random walk. The slope enters quadratically, so halving it quarters the direct information the measurement equation contributes about the latent state.

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The result is analytical and holds independently of any policy episode. Its practical importance is an empirical question, and inflation targeting supplies a natural setting in which to answer it, because a regime that stabilises inflation is expected to weaken the very comovement the NAIRU is recovered from and hence to flatten the estimated curve (McLeay and Tenreyro, 2020). Canada announced an inflation-control target in February 1991 and has maintained it since, giving a date fixed independently of the estimation sample. Across that break Λ deteriorates by a factor of twenty-three, the model becomes unable to sign the unemployment gap in all but a handful of quarters, and the sign restriction that makes the estimated state a NAIRU ceases to be established by the data.

2 Identification of the NAIRU path

Let π_t denote inflation and u_t the unemployment rate. Imposing the accelerationist restriction $\sum_{j=1}^k w_j = 1$ on $\pi_t = \sum_j w_j \pi_{t-j} + \gamma_{2,t}(u_t - u_t^*) + \varepsilon_t$ gives the equivalent reparameterisation

$$\Delta\pi_t = \sum_{i=1}^{k-1} c_i \Delta\pi_{t-i} + \gamma_{2,t}(u_t - u_t^*) + \varepsilon_t, \quad \varepsilon_t \sim N(0, R^{-1}), \quad (1)$$

with $c_i = -\sum_{j>i} w_j$, and the state follows $u_t^* = u_{t-1}^* + \eta_t$, $\eta_t \sim N(0, Q^{-1})$.

Write $u^* = (u_1^*, \dots, u_T^*)'$ and $H = I_T - L$, so that $Hu^* = (u_1^*, \Delta u_2^*, \dots, \Delta u_T^*)'$. With Q_1 the initial-state precision and $D = \text{diag}(Q_1, Q, \dots, Q)$, the state equation contributes the prior precision $K = H'DH$, that is

$$K = \begin{pmatrix} Q_1 + Q & -Q & 0 & \cdots & 0 \\ -Q & 2Q & -Q & \ddots & \vdots \\ 0 & -Q & 2Q & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & -Q \\ 0 & \cdots & 0 & -Q & Q \end{pmatrix}. \quad (2)$$

The interior value $2Q$ is exact: each u_t^* , $1 < t < T$, enters two increments and each contributes Q .

Let $G = \text{diag}(\gamma_{2,1}, \dots, \gamma_{2,T})$ and collect the observed terms of (1) in $\tilde{y}_t = \Delta\pi_t - \sum_i c_i \Delta\pi_{t-i} - \gamma_{2,t}u_t$, so that $\tilde{y} = -Gu^* + \varepsilon$ with $\varepsilon \sim N(0, R^{-1}I_T)$. The likelihood kernel $\exp[-\frac{R}{2}(\tilde{y} + Gu^*)'(\tilde{y} + Gu^*)]$ is quadratic in u^* with second-order term $RG'G$, so

$$K^{\text{post}} = H'DH + RG'G. \quad (3)$$

Since G is diagonal, $RG'G = R \text{diag}(\gamma_{2,1}^2, \dots, \gamma_{2,T}^2)$ and

$$K^{\text{post}} = \begin{pmatrix} Q_1 + Q + \gamma_{2,1}^2 R & -Q & \cdots & 0 \\ -Q & 2Q + \gamma_{2,2}^2 R & \ddots & \vdots \\ \vdots & \ddots & \ddots & -Q \\ 0 & \cdots & -Q & Q + \gamma_{2,T}^2 R \end{pmatrix}. \quad (4)$$

The underlying precision decomposition is standard. To our knowledge, however, it has not previously been used to derive an explicit closed-form diagnostic linking NAIRU identification to the Phillips-curve slope. For interior dates, equation (4) shows that the state equation contributes $2Q$ to the diagonal and observation t contributes $\gamma_{2,t}^2 R$, giving

$$\Lambda_t = \frac{\gamma_{2,t}^2 R}{2Q} = \frac{1}{2} \gamma_{2,t}^2 \left(\frac{\sigma_\eta}{\sigma_\varepsilon} \right)^2. \quad (5)$$

where the second equality uses $R = \sigma_\varepsilon^{-2}$ and $Q = \sigma_\eta^{-2}$. Three properties follow. Λ_t is a per-period quantity, so it does not mechanically improve with sample length. It is proportional to the squared slope, so a flatter curve degrades identification quadratically, and $\Lambda_t \rightarrow 0$ as $\gamma_{2,t} \rightarrow 0$, at which point inference about u_t^* travels only through the state equation and neighbouring observations. And the right-hand expression shows Λ to be the signal-to-noise ratio of Kuttner (1994) and Staiger et al. (1997) weighted by the squared loading: two models with identical signal-to-noise ratios can carry very different amounts of information about the NAIRU.

3 Canadian evidence

Inflation is the annualised quarterly log change in the Canadian CPI excluding food and energy (Statistics Canada table 18-10-0004-01); unemployment is the seasonally adjusted rate for those aged 15 and over (table 14-10-0287-01). Following standard practice (Gordon, 2013; Ball and Mazumder, 2019) we use non-overlapping quarterly rates. The sample runs from 1977Q2 to 2026Q2, $T = 197$. We estimate (1)–(3) with $k = 4$ and regime-specific slopes by Gibbs sampling, drawing $u_{1:T}^*$ in one block by the sparse-precision method of Chan and Jeliazkov (2009). Priors are uninformative, $\gamma_2^{(r)} \sim N(0, 10)$, since the sign and magnitude of the slope are the objects of interest.

Table 1 reports the results. The slope falls from -0.402 (0.181) to -0.072 (0.060), with $P(\gamma_2^{\text{post}} > \gamma_2^{\text{pre}} \mid \text{data}) = 0.981$. The effect on identification is larger than the change in the slope, as (5) implies: Λ falls from 8.8×10^{-4} to 3.8×10^{-5} , a factor of twenty-three.

That deterioration is the slope and nothing else. Since $\Lambda = \gamma_2^2 R / 2Q$ decomposes into three posterior moments, $E[R]$ differs by one per cent across regimes and $E[Q]$ by three, while $E[\gamma_2^2]$ falls by a factor of 22.1. The inflation equation fits equally well in both regimes and the state prior is effectively identical.

Figure 1 shows the consequence. The 90 per cent band widens from ± 1.5 to ± 4.9 percentage points on a sample more than twice as long: the additional observations do not compensate for the weaker loading. In the quantity policy uses, the credible interval for the unemployment gap contains zero in 97 per cent of post-1991 quarters, against 73 per cent before; for all but four quarters since 1991 the model cannot say whether the economy is above or below its NAIRU.

Panel (b) shows why this is more than a loss of precision. The latent trend is a NAIRU only if $\gamma_2 < 0$; applied work generally imposes that restriction rather than testing it. Left free, the pre-1991 posterior gives $P(\gamma_2 < 0 \mid \text{data}) = 0.99$, the post-1991 posterior only 0.778.

Table 1: Phillips curve slope and NAIRU identification

	1977–1990	1991–2026	Difference
γ_2	-0.4018 (0.1814)	-0.0720 (0.0601)	0.3440 (0.1749)
90% credible interval	[-0.709, -0.106]	[-0.190, -0.005]	[0.063, 0.637]
σ_ε	1.623	1.596	
σ_η	0.1621	0.1597	
Mean NAIRU (%)	8.55	6.95	
Λ	8.8×10^{-4}	3.8×10^{-5}	
Observations	55	142	197
$P(\gamma_2^{\text{post}} > \gamma_2^{\text{pre}} \mid \text{data}) = 0.981$			
Without sign restriction: $\gamma_2^{\text{post}} = -0.0497$ (0.0680), $P(\gamma_2^{\text{post}} < 0) = 0.778$			

Notes: quarterly data, CPI excluding food and energy. Four lags of inflation with coefficients constrained to sum to one. Posterior means, standard deviations in parentheses. The ratio compares the per-period likelihood contribution $\gamma_2^2 R$ to the state prior precision $2Q$.

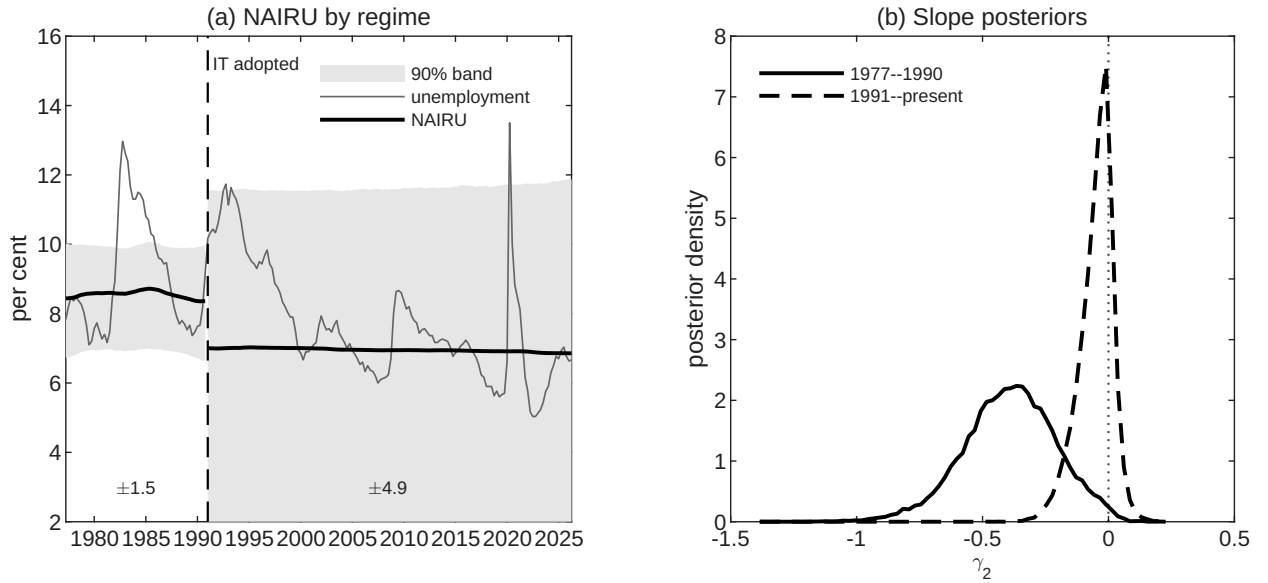


Figure 1: Panel (a): posterior mean NAIRU by regime with 90 per cent credible bands and the observed unemployment rate; annotations give the average half-width of the band. Panel (b): posterior densities of the slope, estimated without the sign restriction.

4 Robustness

Table 2: Robustness

Specification	γ_2		Λ		$P(\gamma_2^{post} < 0)$
	pre	post	pre	post	
twelve-month inflation	-0.079	-0.026	4.3×10^{-4}	5.6×10^{-5}	0.832
CPI all-items	-0.346	-0.076	6.6×10^{-4}	2.1×10^{-5}	0.648
quarterly, 1 lag	-0.123	-0.060	6.9×10^{-5}	1.8×10^{-5}	0.592
quarterly, 2 lags	-0.170	-0.056	1.5×10^{-4}	1.7×10^{-5}	0.593
quarterly, 8 lags	-0.316	-0.122	5.6×10^{-4}	1.1×10^{-4}	0.915
monthly, 12 lags	-0.115	-0.028	2.1×10^{-5}	1.5×10^{-6}	0.782
prior $\nu_q = 200$	-0.394	-0.073	8.2×10^{-4}	3.9×10^{-5}	0.778
prior $\nu_q = 5000$	-0.395	-0.073	8.2×10^{-4}	4.0×10^{-5}	0.777
seed = 99	-0.397	-0.072	8.2×10^{-4}	3.7×10^{-5}	0.781
excl. 2020	-0.397	-0.059	8.4×10^{-4}	2.8×10^{-5}	0.671
prior $\text{var}(\gamma_2) = 1$	-0.379	-0.049	7.8×10^{-4}	3.1×10^{-5}	0.774
prior $\text{var}(\gamma_2) = 100$	-0.393	-0.049	8.4×10^{-4}	3.2×10^{-5}	0.774

Notes: sign restriction lifted throughout; the baseline is quarterly annualised core CPI inflation with four lags. Λ is defined in equation (5).

Table 2 reports twelve alternative specifications, with the sign restriction lifted throughout. Two features are stable. Λ deteriorates across the break in every one, by a factor of at least 3.9 and by twenty or more in seven of the twelve. And $P(\gamma_2^{post} < 0)$ ranges from 0.592 to 0.915 without reaching 0.95, against 0.99 before 1991. The pattern survives changes in the inflation measure, the price index, the frequency, the lag order, the strength of both priors, the random seed, and the exclusion of 2020; setting the prior variance on γ_2 to 1 or 100 rather than 10 moves $P(\gamma_2^{post} < 0)$ from 0.778 to 0.774. Four lags is the shortest specification leaving the quarterly residuals uncorrelated, which is why it is the baseline; the posterior probability of a slope difference, residual diagnostics and MCMC output are in the online appendix.

5 Discussion

Equation (3) holds for any state-space NAIRU model, whatever made the loading small: a smooth posterior path is not evidence that the data locate the NAIRU. Whether the Canadian slope flattened because of inflation targeting, anchored expectations or structural change is a separate question on which the identification result does not depend.

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