

When Tariffs Meet Oil: Optimal Monetary Policy with Household Debt*

Guy Arnold Djolaud

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Abstract

How should a commodity exporter set monetary policy when foreign tariffs and oil prices rise together, and who bears the adjustment? I study this question in a small open economy with nominal household debt, housing collateral, and gradual mortgage repricing, calibrated to Canadian data. Under a CPI rule, the policy rate rises and borrowers absorb most of the contraction: their consumption falls by 1.1 percent on impact, against 0.25 percent for savers. The Ramsey allocation cuts the rate by 14 basis points, roughly halves the output decline, and keeps borrower consumption at its no-shock level on impact, at the cost of higher consumer price inflation. The policy choice is mainly redistributive: relative to the CPI rule, borrowers gain the equivalent of 0.11 percent of permanent consumption, savers lose 0.02 percent, and the aggregate gain is negligible. Faster mortgage repricing lets a smaller rate cut deliver larger immediate relief to indebted households.

JEL codes: E21, E52, F13, F41, Q43.

Keywords: Optimal monetary policy; tariffs; oil prices; household debt; collateral.

I Introduction

A foreign tariff and an increase in oil prices send conflicting signals to the central bank of a commodity exporter. The tariff reduces demand for domestic production. Higher oil prices raise the cost of consumption and intermediate inputs but also increase the value of export receipts. Consumer prices can rise while domestic demand and production weaken. When households carry nominal debt secured by housing, the policy response also decides how this adjustment is shared: it changes the real value of liabilities, the cost of servicing them, and the collateral available to borrowers. Which inflation should policy accommodate, and who absorbs the shock?

This paper studies optimal monetary policy, and its distributional consequences, when a deterioration in foreign market access coincides with a higher oil price. Canada is a natural setting. Its exports are concentrated on one destination, it is a net oil exporter, and the interest cost of its household mortgage debt adjusts gradually to the policy rate. The gains and losses from such an episode fall on different households. Higher oil receipts accrue to the owners of oil income, while higher fuel bills, weaker labor income, and debt service fall on households who may own none of it. A policy-rate cut, in turn, reaches an existing mortgage balance only as that balance reprices.

*Djolaud: Department of Economics, McGill University, 855 Sherbrooke Street West, Montreal, QC H3A 2T7, Canada (email: kili.djolaud@mail.mcgill.ca). The companion replication package contains the archived public data, model code, and validation scripts.

I develop a two-household small open economy with nominal mortgage debt, a housing collateral constraint, and sticky domestic prices. Savers hold financial assets and receive domestic profits and resident oil income. Borrowers finance housing and consumption subject to a collateral ceiling. Oil enters household expenditure and domestic production, and a fixed oil endowment generates export income. A foreign tariff reduces demand for non-oil exports, and a Canadian counter-tariff raises the price of imported consumption goods. The central bank chooses the short nominal rate under commitment. The rate on outstanding household debt follows its own law of motion, so that the speed and the amplitude of mortgage repricing can differ.

Canadian data discipline three margins of the model. First, rates on outstanding mortgage balances respond more gradually than rates on funds advanced. A quarterly projection over 2016Q4–2025Q4 yields an adjustment coefficient of 0.327 and a long-run amplitude of 0.500; I treat this law of motion as invariant across policy regimes and report results along its estimated covariance. Second, national accounts show that real production and real domestic income diverge substantially during large terms-of-trade movements, so that aggregate trading gains and household purchasing power must be tracked separately. Third, local projections on the oil-supply-news shocks of Känzig (2021) show that the appreciation of the Canadian dollar after oil news more than offsets, in Canadian dollars, the rise in the foreign-currency price of imported consumer goods.

The main experiment combines a 5 percent foreign tariff, a 1.5 percent Canadian counter-tariff, and a 20 percent increase in the oil price, all unanticipated and gradually decaying. Every policy starts from the same physical state and the same inherited commitments, and each shock response is measured against that policy's own no-shock transition. I compare the Ramsey allocation with an inertial Taylor rule on consumer price inflation (the CPI rule), strict stabilization of domestic producer prices (domestic price stability), and a CPI rule whose two coefficients are optimized for the joint episode.

The first finding is that the policy regime largely determines who bears the joint shock. Under the CPI rule, the policy rate rises by 33 basis points and real GDP falls by 1.03 percent on impact. Borrowers absorb most of the contraction: their consumption falls by 1.13 percent, more than four times the 0.25 percent decline for savers. The Ramsey allocation instead cuts the rate by 14 basis points and limits the output decline to 0.49 percent. Borrower consumption rises slightly on impact, by 0.06 percent, and saver consumption is essentially unchanged. The cost is higher headline inflation: CPI inflation rises by 2.02 annualized percentage points rather than 1.10, while domestic producer inflation rises by only 0.12 point. Ramsey accepts the relative-price component of inflation to protect domestic production and the households most exposed to it. Most of the initial accommodation comes from the tariff; the oil shock alone calls for a cut of about 2 basis points.

The second finding is that the welfare stakes are mainly distributional. Relative to the CPI rule, Ramsey raises borrowers' welfare by the equivalent of 0.112 percent of permanent consumption and lowers savers' welfare by 0.018 percent, while the aggregate gain is only 0.0022 percent. The choice between these policies is therefore less a question of aggregate efficiency than of how the burden is shared. The episode itself is also unevenly distributed. Because savers own the oil income, they gain from the joint shock under both policies, whereas borrowers lose under both. The CPI rule makes the borrowers' loss from the episode roughly ten times larger than under Ramsey.

The third finding identifies the channels behind this redistribution. A smaller decline in labor income supports borrowers, unexpected CPI inflation reduces the real value of their inherited nominal debt, and the path of the policy rate feeds into future mortgage payments. Timing matters: debt service at impact depends

on the inherited mortgage rate, so the initial consumption response does not come from an immediate cut in interest payments. Indexation isolates the debt-revaluation channel: when surprise inflation no longer erodes repayments, the fall in borrower consumption under the CPI rule deepens from 1.18 to 1.32 percent in a first-order diagnostic. House prices barely move under Ramsey, and freezing the collateral price changes little; the support to borrowers works mainly through income and debt revaluation rather than collateral values.

The fourth finding is that mortgage repricing changes how much rate movement this redistribution requires. Relative to a slow, unit-amplitude calibration, the estimated repricing law reduces the initial Ramsey cut from 26 to 14 basis points, yet the mortgage rate falls more on impact, by 2.3 rather than 1.3 basis points, because adjustment is faster. The policy rate is thus an incomplete measure of the relief reaching indebted households.

The incidence of accommodation also depends on who owns the oil income and on energy exposure. The direction of the Ramsey response holds across nine selected calibrations, and Ramsey still eases when borrowers become a negligible share of the economy: debt shapes the strength and the incidence of the response rather than its sign. Borrowers' initial consumption response, by contrast, turns negative when half of oil income accrues abroad, when the oil shock is less persistent, under supply–use energy shares, or with a higher foreign cost of imported goods. Lowering the policy rate does not by itself guarantee that indebted households are protected.

The fifth finding concerns implementable rules. Domestic price stability delivers about 37 percent of borrowers' Ramsey gain and closes 55 percent of the gap to Ramsey in the commitment criterion.¹ Re-optimizing the two coefficients of the CPI rule closes only 6 percent of that gap: it raises the initial rate by 96 basis points and deepens the fall in borrower consumption to 1.61 percent. In this experiment, which inflation measure enters the rule matters more for incidence than how strongly the rule responds.

The model also yields an analytical result on imported inflation. With logarithmic utility and Cobb–Douglas consumption, an increase in the relative foreign price of the imported consumption basket admits an exact transformation of the equilibrium and planner conditions. It lowers composite purchasing power and import volumes for both households, but leaves the Ramsey interest-rate path and the domestic production, labor, and housing allocations unchanged. An import cost can therefore lower households' purchasing power, and in the calibrated case turn borrowers' impact consumption response negative, without changing the optimal policy rate. A CPI rule does not share this invariance, because the transformed CPI inflation rate enters its feedback directly.

Related literature. The analysis draws on the small open economy framework of Galí and Monacelli (2005), in which the distinction between domestic and consumer prices is central to the policy problem. Bergin and Corsetti (2023) study the monetary stabilization of tariff shocks, and Drechsel et al. (2026) show how commodity exposure, production inputs, and financial conditions shape optimal responses to commodity prices. Relative to these papers, I bring tariff and oil exposure into one economy with household balance sheets and ask who bears the adjustment under alternative policies, and how mortgage repricing and the ownership of external income change the answer.

The household structure builds on the link between nominal borrowing, housing collateral, and monetary transmission in Iacoviello (2005). The redistribution channel of Auclert (2019), the heterogeneous-household

¹The commitment criterion includes the value of inherited promises. Section V reports it separately from household welfare.

transmission mechanisms of Kaplan et al. (2018), and the evidence in Cloyne et al. (2020) motivate the separation of asset holders from indebted households. The two-type structure keeps the contractual interest-rate margin and the collateral condition transparent, and it allows the same external shock to be traced through borrower budgets, saver income, and the planner’s commitment conditions.

Finally, Acharya and Challe (2025) study inequality and optimal monetary policy in an open economy, emphasizing when household heterogeneity changes the case for domestic price stability. I focus instead on collateralized nominal debt, gradual repricing, and the ownership of oil income, and I report group-specific welfare alongside the aggregate criterion.

The rest of the paper proceeds as follows. Section II presents the Canadian evidence. Section III describes the economy and the policy problem. Section IV explains the calibration and the experiments. Section V reports the policy, distributional, and welfare results, and Section VI concludes. The appendices provide the full equilibrium system, the empirical methods, the separability proof, and the numerical implementation.

II Canadian Exposure and Empirical Discipline

Canada combines a concentrated export market with substantial commodity exposure. In 2024, the United States purchased 75.9 percent of Canadian domestic merchandise exports, worth C\$547.4 billion (Global Affairs Canada, 2025). The denominator is customs-based domestic exports: the statistic includes energy, excludes re-exports, and is not an export-to-GDP ratio. Canada also exported considerably more crude oil to the United States than it imported from that country (Canada Energy Regulator, 2025). These features create distinct channels through which an external disturbance reaches Canadian households. A tariff changes demand for domestic production; an oil-price movement changes both expenditure costs and the purchasing power of export receipts.

I discipline these channels with three complementary objects: the dynamics of interest rates on outstanding mortgage debt, the divergence between production and real domestic income, and the response of imported-goods prices to identified oil-supply news. The evidence restricts measurement and establishes plausible magnitudes; household incidence and the policy counterfactual are then determined within the model. Table 1 reports the mortgage estimates and fit measures; Table 2 distinguishes parameters informed by these exercises from maintained assumptions.

A Mortgage Rates on the Stock of Debt

For an indebted household, the interest rate on the outstanding balance determines current interest payments. The rate charged on newly advanced funds describes a different margin. Figure 1 compares the policy rate with volume-weighted mortgage rates from the Bank of Canada’s lending statistics (Bank of Canada, 2026a,b). The distinction is visible during the recent tightening cycle. Between 2021Q4 and 2023Q4, the rate on outstanding mortgages rose from approximately 2.35 to 4.49 percent, whereas the rate on advances rose from 1.69 to 6.40 percent. The outstanding rate remained approximately 4.13 percent in 2025Q4, when the policy rate was 2.25 percent. Current policy and the interest burden inherited from earlier lending conditions can therefore move very differently.

Advances include renewals and refinancing as well as newly extended credit. The stock–flow gap reflects portfolio repricing and changing composition, rather than a directly observed contractual reset probability.

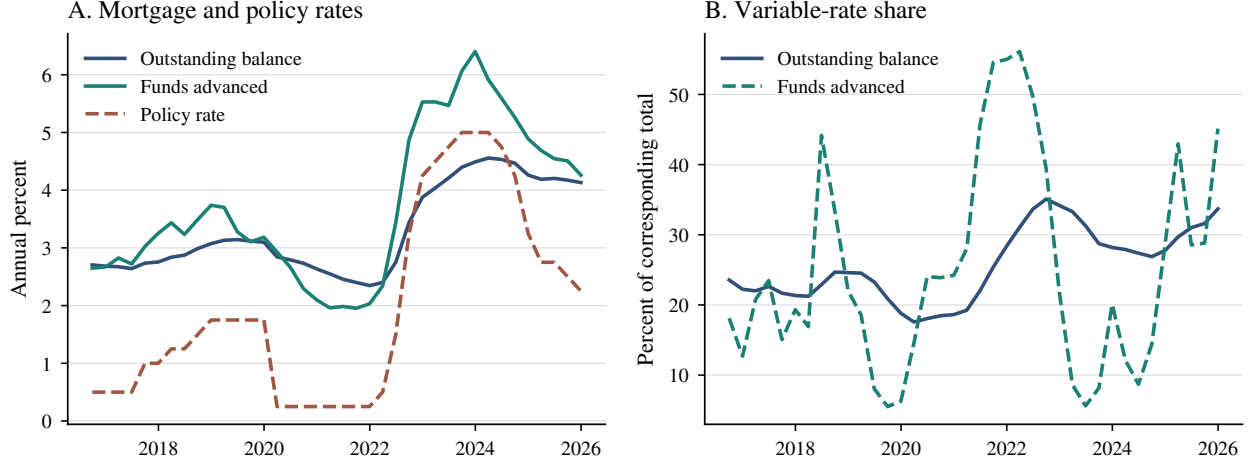


Figure 1. Mortgage Repricing on Outstanding Debt and Funds Advanced

Notes: Panel A reports annual interest rates at the end of each quarter. Insured and uninsured mortgage rates are weighted by their corresponding balances or advances. Panel B reports the variable-rate share of each total. Funds advanced include renewals and refinancing. The sample runs from 2016Q3 to 2025Q4. Sources: Bank of Canada lending rates and volumes (Bank of Canada, 2026a,b); author's calculations.

This focus on the outstanding portfolio is also present in ToTEM III, where new lending and contract renegotiation jointly update the rate on household debt (Corrigan et al., 2021). Appendix B.2 details the aggregation and the institutional comparison.

To summarize the portfolio dynamics at the model's quarterly frequency, I estimate

$$r_t^m = c + ar_{t-1}^m + br_t + u_t, \quad (1)$$

where r_t^m and r_t are the mortgage and policy rates expressed in quarterly units. End-of-quarter observations provide 37 usable observations from 2016Q4 through 2025Q4. Reparameterizing the projection gives an adjustment coefficient $\rho = 1 - a$ and an amplitude $\kappa_m = b/(1 - a)$. The estimates are $\rho = 0.327$ and $\kappa_m = 0.500$, with marginal 95 percent HAC intervals of $[0.151, 0.502]$ and $[0.406, 0.594]$. Imposing unit amplitude, $a + b = 1$, is rejected in this sample, with a p -value of 0.005.

Separating speed from amplitude also improves the conditional recursive fit. Estimating coefficients through 2022Q4 and conditioning on the subsequently observed policy rate yields a 2023–2025 root mean squared error of 0.266 annual percentage points, compared with 1.215 under unit amplitude. At a one-quarter horizon, however, the unrestricted projection has an error of 0.161 percentage points, exceeding the random-walk benchmark of 0.115. The projection therefore does not dominate at all horizons.

Equation (1) is a descriptive projection, not a causal estimate of monetary transmission. Policy rates are endogenous, fixed mortgage rates also depend on longer yields and spreads, and residual persistence remains. The coefficients discipline a parsimonious law of motion whose invariance across policy regimes is an assumption of the policy experiment. The emphasis on mortgage status is consistent with the heterogeneous consumption responses documented by Cloyne et al. (2020); here the aggregate exercise measures the interest-rate margin.

Table 1. Mortgage Repricing: Estimates and Conditional Fit

<i>A. Dynamic projection, 2016Q4–2025Q4</i>			
Object	Estimate	95 percent interval	Observations
Adjustment speed, ρ	0.3268	[0.1512, 0.5024]	37
Pass-through amplitude, κ_m	0.5000	[0.4057, 0.5942]	37
Impact response, $\rho \kappa_m$	0.1634	[0.0919, 0.2348]	37
<i>B. Conditional 2023–2025 RMSE (annual percentage points)</i>			
Projection	Recursive	One quarter	Observations
Speed and amplitude	0.2665	0.1613	12
Unit-amplitude restriction	1.2151	0.3247	12
Random walk	0.4592	0.1152	12

Notes: End-of-quarter observations, 2016Q4–2025Q4. Rates on insured and uninsured outstanding mortgages are weighted by their respective balances, then converted to quarterly gross rates. The ARX is $R_t^m = c + aR_{t-1}^m + bR_t + u_t$, with $\rho = 1 - a$ and $\kappa_m = b/(1 - a)$. Intervals use HAC inference with four lags, Student- t critical values, and the delta method where needed. The restriction $a + b = 1$ has $p = 0.00496$. These are descriptive projections, not identified monetary-policy shock responses. Panel B fixes coefficients estimated through 2022Q4 and conditions on the realized policy-rate path. Recursive predictions recycle their own lagged mortgage rate; one-quarter predictions use its observed lag.

B Oil Prices, Production, and Purchasing Power

Output volume does not fully describe the resources available for domestic expenditure in a commodity exporter. Changes in export prices relative to import prices alter the purchasing power of a given quantity of production. Figure 2 therefore reports real GDP, real gross domestic income, and the terms of trade from the Canadian national accounts (Statistics Canada, 2026). Real gross domestic income incorporates trading gains and losses.

The oil-price decline from 2014Q2 to 2016Q1 illustrates the distinction. Over that interval, real GDP increased by approximately 1.9 percent, while real gross domestic income fell by 2.1 percent and the terms of trade declined by 11.3 percent. The episode from 2019Q4 to 2022Q4 shows the opposite ordering: production rose by about 5.4 percent, real income by 8.1 percent, and the terms of trade by 8.8 percent. These changes are calculated from the published quarterly indexes. The episodes contain multiple disturbances, but their accounting implication is direct: production and purchasing power need not move together.

For the policy problem, this distinction separates an aggregate trading gain from its distribution. A higher oil price can raise the purchasing power of export receipts while increasing households' fuel expenditure and firms' input costs. Whether constrained borrowers share in the income gain depends on ownership, earnings, and transfers. Oil-income incidence across model households is an explicit assumption: national accounts establish the aggregate trading gain but not its distribution. This separation connects the analysis to the role of heterogeneous interest-rate and income exposures in monetary transmission (Auclert, 2019).

C Oil News, the Exchange Rate, and Imported-Goods Prices

An oil-price increase need not raise every import price measured in Canadian dollars. The exchange-rate response can offset a change in the foreign-currency price. To distinguish these margins, I use the oil-supply-news shock series of Känzig (2021) and estimate quarterly local projections following Jordà (2005). The regressor is the structural shock recovered from the author's identified VAR, rather than the raw high-

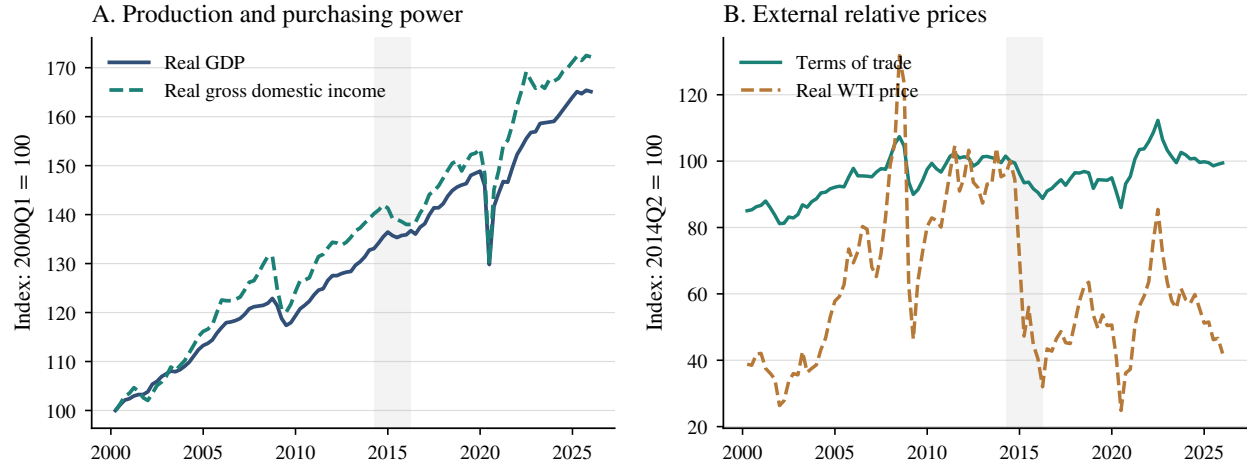


Figure 2. Production, Domestic Purchasing Power, and Oil Prices

Notes: Real GDP and real gross domestic income are indexed to 100 in 2000Q1. The terms of trade and the WTI price deflated by the US consumer price index are indexed to 100 in 2014Q2. Shading marks 2014Q2–2016Q1. The series are quarterly. Real gross domestic income incorporates trading gains and losses. Sources: Statistics Canada (Statistics Canada, 2026), the public oil-price and US price series documented in Appendix B.3, and author’s calculations.

frequency surprise used as an instrument in that VAR. Responses are normalized to a ten-log-point increase in real WTI at impact.

Appendix Figure B.1 reports responses of the exchange rate, an imported-consumer-goods deflator, and the corresponding foreign-currency prices. The deflator’s effective estimation window is 1990Q4–2018Q2 (Statistics Canada, 2018). It covers consumer goods, with energy classified separately, rather than all non-energy imports. Converting it to US dollars and dividing by US consumer prices gives a proxy for the foreign relative price relevant to the model. Appendix B.3 describes the construction and inference.

At impact, the Canadian dollar appreciates: Canadian dollars per US dollar decline by 1.50 log points, with a 95 percent interval of $[-2.11, -0.88]$. The consumer-import deflator falls by 0.99 log points in Canadian dollars, with interval $[-1.53, -0.46]$. The same price rises by 0.51 log points after conversion to US dollars. Relative to the US consumer price index, its increase is smaller, at 0.23 log points, with interval $[-0.06, 0.52]$. The positive nominal foreign-currency response therefore does not by itself measure the foreign relative-cost disturbance entering the model.

The ratio of the last response to the contemporaneous oil-price response is 0.023, with interval $[-0.006, 0.052]$. I set the model’s import-cost elasticity, γ_f , to zero in the reference calibration and examine $\gamma_f = 0.05$, close to the upper endpoint, as a positive-cost scenario. For a 20 percent oil-price increase and the reference import weight, its direct contribution to the consumer price index is approximately 0.144 log points before exchange-rate and policy adjustments. The model determines the equilibrium consequences. Mapping the historical projections into immediate transmission with common persistence remains a modeling restriction.

D Expenditure Shares and the Calibration

Supply and use accounts provide an additional check on the size of the direct and production-cost channels (Statistics Canada, 2025). Pooling nominal flows over 2017–2019, household expenditure on refined

petroleum products is 2.10 percent of GDP and 3.75 percent of household consumption. Industry use is 3.35 percent of GDP. The consumption denominator is the relevant starting point for a household expenditure weight; the production denominator is relevant for intermediate inputs. Neither ratio is itself a crude-oil share, since purchaser prices include refining, distribution, and taxes.

These accounts underpin an alternative calibration. The crude-equivalent conversions are accounting assumptions, and the all-industry numerator is broader than the model's non-oil producer. The reference retains a direct oil-consumption weight of 1.2 percent, with the expenditure-based alternative reported separately. In the model without investment or government absorption, steady-state consumption equals GDP; this normalization prevents an exact match to all observed expenditure ratios simultaneously.

III Model

The economy combines three features that matter for monetary policy in Canada: foreign demand for domestic production, income from a net oil endowment, and household borrowing secured by housing. Domestic producers set prices subject to adjustment costs. Households differ in patience and balance sheets, while the effective borrowing rate adjusts gradually to the policy rate. This structure connects the open-economy stabilization problem of Galí and Monacelli (2005) to the collateral mechanism in Iacoviello (2005). Its two household types provide a tractable account of the redistribution emphasized by Auclert (2019). Time is quarterly. The experiments study deterministic equilibrium paths following an initial tariff innovation and an oil-price innovation; the expectation operators below describe the underlying equilibrium conditions.

The central policy trade-off arises because the two shocks affect different relative prices. A tariff weakens the demand for domestic output at a given exchange rate. More expensive oil raises consumption and production costs, but also the value of the country's oil income. The exchange rate links these external disturbances to domestic inflation and competitiveness. Household debt adds two distinct transmission margins: the real burden of outstanding nominal claims and the response of future borrowing costs and collateral values. The model keeps these margins within a single flow-of-funds system, so that a gain to a debtor has an explicit counterpart elsewhere in the economy.

A Goods, Prices, and External Shocks

Households consume a domestic good, an imported non-oil composite, and oil, indexed by $j \in \{h, f, o\}$. The common consumption aggregator is CES with elasticity η and weights $\omega_j > 0$ summing to one. The baseline sets $\eta = 1$, giving Cobb–Douglas expenditure shares. Let $P_{C,t}$ denote its price index and write $p_{j,t} = P_{j,t}/P_{C,t}$. Aggregate consumption is $C_t = (1 - \lambda)C_{S,t} + \lambda C_{B,t}$. Goods demands and the price-index restriction are

$$C_{j,t} = \omega_j p_{j,t}^{-\eta} C_t, \quad \sum_j \omega_j p_{j,t}^{1-\eta} = 1, \quad (2)$$

with the second expression replaced by $\sum_j \omega_j \log p_{j,t} = 0$ when $\eta = 1$.

Let E_t be the domestic-currency price of foreign currency and P_t^* the foreign numeraire. The real exchange rate is $s_t = E_t P_t^*/P_{C,t}$; an increase is a real depreciation. I normalize the fixed foreign numeraire to $P_t^* = 1$. This numeraire is distinct from the relative foreign price of the particular import bundle purchased by Canada.

Tariff and oil states follow

$$x_t^\tau = \rho_\tau x_{t-1}^\tau + \varepsilon_t^\tau, \quad x_t^o = \rho_o x_{t-1}^o + \varepsilon_t^o. \quad (3)$$

The US tariff factor is $\exp(x_t^\tau)$, the Canadian tariff factor is $T_t^{CA} = \exp(\chi_{CA} x_t^\tau)$, and the world oil price is $P_{o,t}^* = \bar{P}_o^* \exp(x_t^o)$. Prices faced by Canadian purchasers satisfy

$$p_{o,t} = s_t P_{o,t}^*, \quad v_{f,t} = \exp(\gamma_f x_t^o), \quad p_{f,t}^b = s_t v_{f,t}, \quad p_{f,t} = T_t^{CA} p_{f,t}^b. \quad (4)$$

Here p_f^b is the price at the border. The parameter γ_f allows the oil shock to raise foreign costs embodied in the non-oil import composite. This basket excludes oil consumed directly and oil used by domestic firms. Its price does not change the foreign numeraire or foreign competitors' prices. The baseline sets $\gamma_f = 0$; a positive value is examined as a separate cost scenario. Canadian tariff revenue, $\mathcal{T}_t = (T_t^{CA} - 1)p_{f,t}^b C_{f,t}$, is rebated equally per capita.

The distinction between the foreign numeraire and the specific import price also fixes the trade experiment. Exporters compete against an unchanged foreign price index, even in the scenario with $\gamma_f > 0$. If the same price increase applied to their competitors, it would also enter export demand and define a different experiment. Likewise, the Canadian tariff is applied to the modeled imported composite, whereas the US tariff enters the demand for modeled exports. The two factors describe the exposure of these baskets, rather than an assumption that every international transaction faces an identical statutory rate.

B Households, Collateral, and Borrowing Rates

A share $1 - \lambda$ of households are savers, S , and a share λ are borrowers, B . Type i maximizes

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta_i^t \left[u_\sigma(C_{i,t}) + \xi_i \log H_{i,t} - \chi_i \frac{L_{i,t}^{1+\psi}}{1+\psi} \right], \quad \beta_B < \beta_S, \quad (5)$$

where $u'_\sigma(C) = C^{-1/\sigma}$ and the baseline uses $u_1(C) = \log C$. Housing supplies services and collateral but is in fixed aggregate supply: $(1 - \lambda)H_{S,t} + \lambda H_{B,t} = \bar{H}$. Write $q_t = Q_t/P_{C,t}$ and $w_t = W_t/P_{C,t}$ for its price and the wage in consumption units. Gross quarterly CPI and producer inflation are $\Pi_{C,t} = 1 + \pi_{C,t}$ and $\Pi_{h,t} = 1 + \pi_{h,t}$.

Let $b_t = B_t/P_{C,t}$ be end-of-period debt per borrower, where B_t is a nominal promise issued at date t and repaid at $t + 1$. The borrower budget and borrowing limit are

$$C_{B,t} + q_t(H_{B,t} - H_{B,t-1}) + \frac{R_{t-1}^m b_{t-1}}{\Pi_{C,t}} = w_t L_{B,t} + b_t + \mathcal{T}_t, \quad (6)$$

$$b_t \leq m q_t H_{B,t}. \quad (7)$$

The limit applies to the current house value. All reported policy comparisons are solved in its binding region, with a nonnegative multiplier μ_t . This restriction links housing valuations to borrowing capacity. Inflation also changes the purchasing power transferred by previously contracted nominal debt, whereas changes in the current borrowing rate affect the cost of debt issued today.

The timing is consequential for the initial response. At the moment the shock arrives, R_{t-1}^m and nominal debt issued in the previous quarter are inherited. Monetary policy can change the real value of that repayment

through current inflation, but a change in R_t^m first enters contractual debt service next quarter. Consumption can nevertheless respond immediately because households anticipate future financing conditions and trade housing at the current price. A lower q_t simultaneously reduces the value of housing wealth and the collateral available for new borrowing. These effects are endogenous outcomes of the housing asset conditions, rather than a separate rule relating consumption mechanically to house prices.

With $\Lambda_{i,t} = C_{i,t}^{-1/\sigma}$, the two borrower asset conditions are

$$\Lambda_{B,t} = \beta_B R_t^m \mathbb{E}_t \left[\frac{\Lambda_{B,t+1}}{\Pi_{C,t+1}} \right] + \mu_t, \quad (8)$$

$$q_t \Lambda_{B,t} = \frac{\xi_B}{H_{B,t}} + m q_t \mu_t + \beta_B \mathbb{E}_t [\Lambda_{B,t+1} q_{t+1}]. \quad (9)$$

Housing provides both services and additional collateral. The labor condition for either type is $w_t \Lambda_{i,t} = \chi_i L_{i,t}^\psi$. Savers have the corresponding unconstrained housing condition and price one-period deposits paying the policy rate R_t .

Financial intermediaries fund loans with saver deposits. Market clearing requires deposits per saver to equal $a_t = \lambda b_t / (1 - \lambda)$. The effective borrower rate follows

$$R_t^m - \bar{R} = (1 - \rho)(R_{t-1}^m - \bar{R}) + \rho \kappa_m (R_t - \bar{R}). \quad (10)$$

The speed ρ and long-run marginal response κ_m are separate parameters. The centered specification preserves $R^m = R = \bar{R}$ at the zero-inflation reference allocation. Intermediary profits or losses, $\lambda (R_{t-1}^m - R_{t-1}) b_{t-1} / \Pi_{C,t}$, accrue to savers. Thus slow repricing changes household cash flows without creating an unrecorded resource transfer. Equation (10) is a reduced-form description of the effective rate on debt rolled over each quarter; it does not introduce mortgage vintages or a contractual amortization schedule. Its use across policy regimes maintains the estimated rate law.

Savers also hold one-period foreign assets f_t , measured per saver in the fixed foreign numeraire, paying the constant gross return R^* . Acquisition costs are $g(f_t) = \psi_F (f_t - \bar{f})^2 / (2\bar{V})$ in the same units. This portfolio cost provides a stationary external-asset closure (Schmitt-Grohé and Uribe, 2003). Optimal holdings satisfy

$$s_t \Lambda_{S,t} [1 + g'(f_t)] = \beta_S R^* \mathbb{E}_t [s_{t+1} \Lambda_{S,t+1}]. \quad (11)$$

The future real exchange rate converts the foreign payoff into consumption units. Savers own domestic firms, intermediaries, and the resident share of the oil endowment. Appendix A gives their budget and the complete equilibrium system.

Ownership therefore matters for the incidence of an oil windfall. Savers receive resident oil income directly. Borrowers share its general-equilibrium effects through wages, goods prices, housing, and tariff rebates. They also bear the financing consequences of their outstanding debt. The economy can experience higher aggregate purchasing power alongside weaker borrower consumption, even though both groups face the same consumption prices and participate in the labor market. This distinction is what the two-household structure contributes beyond the aggregate trade and cost channels.

C Production, Trade, and National Accounts

Monopolistically competitive domestic producers combine labor and oil: $Y_t = AL_t^{1-\alpha_o}M_t^{\alpha_o}$, where $L_t = (1-\lambda)L_{S,t} + \lambda L_{B,t}$. Their marginal cost relative to the producer price is

$$mc_t = \frac{1}{A} \left[\frac{w_t}{(1-\alpha_o)p_{h,t}} \right]^{1-\alpha_o} \left[\frac{p_{o,t}}{\alpha_o p_{h,t}} \right]^{\alpha_o}, \quad M_t = \frac{\alpha_o mc_t p_{h,t} Y_t}{p_{o,t}}. \quad (12)$$

Prices incur Rotemberg (1982) adjustment costs of $\mathcal{D}_t = \phi \pi_{h,t}^2 Y_t / 2$ units of domestic output in symmetric equilibrium. A sales subsidy $\varsigma = 1/(\varepsilon - 1)$ offsets the zero-inflation monopoly markup, where ε is the elasticity of demand across domestic varieties. Savers finance this subsidy through a lump-sum tax. The exact nonlinear Phillips equation, including the quadratic term implied by the cost convention, is reported in the appendix.

The productive oil share α_o enters unit costs as well as input demand. An increase in the oil price relative to domestic output raises marginal cost at a given real wage, and firms substitute away from oil. The consumption weight ω_o instead governs direct exposure in the household basket. Because the two parameters apply to different expenditure bases, equalizing them would impose an additional restriction. The pricing friction turns these relative-cost movements into a stabilization problem: adjusting domestic prices consumes resources, while preventing adjustment changes output and factor demands. Producer-price stabilization consequently has a different allocation from stabilizing the broader consumption index.

Foreign demand depends on the tariff-inclusive price of domestic goods in foreign currency:

$$X_t = \bar{X} \left[e^{x_t^r} \frac{p_{h,t}}{s_t} \right]^{-\eta_X}, \quad Y_t = C_{h,t} + X_t + \mathcal{D}_t. \quad (13)$$

The tariff reduces exports at a given relative price, while depreciation supports competitiveness. Oil supply is a fixed endowment $\bar{Q}_o$, with no extraction investment or variable extraction cost. Its net export volume is $NX_t^o = \bar{Q}_o - C_{o,t} - M_t$. A fraction κ_o of oil income accrues to residents and the remainder is remitted abroad. This ownership split affects national income even though all domestic oil production enters GDP. The baseline has $\kappa_o = 1$.

Let $F_t = (1-\lambda)f_t$ and $G_t = (1-\lambda)g(f_t)$. External accounts obey

$$s_t(F_t + G_t) = s_t R^* F_{t-1} + p_{h,t} X_t + p_{o,t} NX_t^o - p_{f,t}^b C_{f,t} - (1-\kappa_o)p_{o,t} \bar{Q}_o. \quad (14)$$

Import payments use border prices; domestic tariff rebates do not enter foreign receipts. Together with household budgets, this equation closes the economy's flow accounts. There is no investment or government absorption. At zero net foreign assets, full resident oil ownership and the zero-inflation reference allocation, aggregate consumption therefore equals GDP. Calibration respects this accounting restriction.

I distinguish CPI-deflated GDP, $p_{h,t}(Y_t - \mathcal{D}_t) - p_{o,t}M_t + p_{o,t}\bar{Q}_o$, from activity at fixed reference prices:

$$GDP_t^{\text{vol}} = Y_t - \mathcal{D}_t - \bar{P}_o^* M_t + \bar{P}_o^* \bar{Q}_o. \quad (15)$$

The normalization sets the domestic reference price to one. An oil-price increase can raise the valuation of

the endowment while reducing non-oil production; these two measures preserve that distinction.

In particular, the physical contribution of the oil endowment to the volume measure is constant. Changes in that measure arise from domestic production, productive oil use, and the resources absorbed by price adjustment. Changes in CPI-deflated GDP additionally reflect the relative prices at which these flows are valued. The external-income remittance associated with foreign ownership belongs in national income and the balance of payments; it does not remove domestically produced oil from GDP. This accounting separation is useful when an exporter experiences an improvement in its terms of trade at the same time as a decline in non-oil activity.

D Monetary Policy and Welfare

The policy instrument is the gross nominal rate R_t . The benchmark rule, which I call the CPI rule, is a Taylor rule that responds to CPI inflation with interest-rate smoothing:

$$\log \frac{R_t}{\bar{R}} = \rho_R \log \frac{R_{t-1}}{\bar{R}} + (1 - \rho_R) \phi_\pi \log \frac{\Pi_{C,t}}{\bar{\Pi}_C}. \quad (16)$$

Here $\bar{R} = 1/\beta_S$ and $\bar{\Pi}_C = 1$ are the private reference values. Domestic price stability instead imposes $\pi_{h,t} = 0$. Under commitment, the planner chooses the policy path subject to all private equilibrium conditions. Its period objective and discounted utility are

$$U_t = (1 - \lambda) \alpha_S u_{S,t} + \lambda \alpha_B u_{B,t}, \quad W_0 = \sum_{t \geq 0} \beta_S^t U_t. \quad (17)$$

The Pareto weights are fixed inverse marginal utilities at the zero-inflation private reference allocation, normalized so that $(1 - \lambda) \alpha_S + \lambda \alpha_B = 1$. This Negishi normalization equalizes weighted marginal consumption utilities per household at that reference allocation. The planner uses β_S for both types. For distributional reporting, individual values V_i retain each household's own discount factor β_i ; W is consequently not the weighted sum of those two individually discounted values.

I use the timeless commitment formulation of Woodford (1999). Let $g_t(y_{t-1}, y_t, y_{t+1})$ collect the 17 private equilibrium constraints and let ζ_{-1} denote inherited planner multipliers. The associated date-zero criterion is

$$J_0 = W_0 + \beta_S^{-1} \zeta_{-1}^T g_{-1}(y_{-2}, y_{-1}, y_0). \quad (18)$$

The boundary term carries the commitments inherited by the planner into the initial policy choice. Policy comparisons use the same physical prehistory and the same inherited multipliers, taken from the Ramsey steady state. That state need not coincide with the zero-inflation allocation used to calibrate preferences. For each policy, the effect of a shock is measured relative to its own no-shock transition from this common state.

With logarithmic utility, the consumption equivalent of an individual utility difference is $100\{\exp[(1 - \beta_i)\Delta V_i] - 1\}$ percent. For social utility it is $100\{\exp[(1 - \beta_S)\Delta W] - 1\}$, using the normalization of the Pareto weights. I report these measures separately from differences in J : the inherited-commitment term is part of the policy criterion, not household consumption utility. This distinction allows the policy ranking and its distributional consequences to be assessed on their respective terms.

The feasible policy set retains household optimization, the borrowing limit, price setting, and the external

budget. Monetary policy cannot assign the oil windfall directly to borrowers or replace the collateral constraint with state-contingent transfers. It acts through prices, the exchange rate, and the two interest rates within the fiscal and ownership arrangements already specified. Consequently, the optimal response internalizes both aggregate resource costs and distributional effects, evaluated with the stated Pareto weights. The comparison identifies how commitment uses the available monetary margins; it does not attribute to the central bank instruments that belong to a different fiscal or financial-policy problem.

IV Quantitative Strategy

A Calibration and Accounting Targets

The model is quarterly. The calibration combines preference and technology parameters, balance-sheet targets, trade accounting, and the mortgage-rate projection described in Section II. Table 2 reports the central values and the discipline behind each. The distinction matters because a dynamic coefficient inferred from an aggregate rate series is a different object from a balance-sheet target or a maintained preference parameter.

Savers discount at $\beta_S = 0.9975$ and borrowers at $\beta_B = 0.99$. Borrowers represent 30 percent of households, and their collateral parameter is $m = 0.8$. Reference debt is 2.9 times quarterly GDP, equivalent to 0.725 times annual GDP; the housing-value target is 10.2 times quarterly GDP, or 2.55 times annual GDP. These are rounded structural targets. Given the population share and the collateral relation, housing preferences and labor disutility scales are chosen to support the reference allocation. The borrower share is a model partition

Table 2. Quarterly Calibration

Parameter or target	Value	Discipline
Discount factors, β_S, β_B	0.9975, 0.99	Preference calibration
Borrower share, λ	0.30	Household calibration
Loan-to-value parameter, m	0.80	Collateral calibration
Preference parameters, σ, η, ψ	1, 1, 1	Log utility; Cobb–Douglas
Debt / annual GDP	0.725	Rounded balance-sheet target
Housing value / annual GDP	2.55	Rounded balance-sheet target
Price elasticity, ε	6	Markup calibration
Price-adjustment cost, ϕ	71.464	Equivalent Calvo parameter 0.75
Household oil weight, ω_o	0.012	Consumption exposure
Oil production share, α_o	0.01158	Input-expenditure mapping
Foreign consumption weight, ω_f	0.15765	Trade accounting
Non-oil exports / GDP	0.12	Trade calibration
Net crude exports / GDP	0.03765	Trade accounting
Export elasticity, η_X	2	Elasticity calibration
Resident oil ownership, κ_o	1	Endowment-income assignment
Portfolio-cost coefficient, ψ_F	0.001	External-asset stationarity
Mortgage speed, ρ	0.3268	Aggregate rate projection
Mortgage amplitude, κ_m	0.5000	Aggregate rate projection
Import-cost exposure, γ_f	0	Reference; 0.05 alternative
Shock persistence, ρ_τ, ρ_o	0.95, 0.95	Episode specification
CPI-rule coefficients, ρ_R, ϕ_π	0.80, 1.50	Policy comparator

Notes: Reference quarterly GDP and aggregate housing are normalized to one. Debt and housing targets equal 2.9 and 10.2 times quarterly GDP. Preference scales and domestic-good weights are recovered from the stationary equilibrium. The model has no investment or government purchases, so its reference consumption-to-GDP ratio is one. Appendix B documents the energy mappings and empirical parameter construction.

into constrained and unconstrained households, rather than the observed fraction of households holding any mortgage.

Log consumption utility and a Cobb–Douglas expenditure aggregator give unit curvature and unit intratemporal elasticity. The inverse Frisch elasticity is one. An elasticity of substitution of six across differentiated domestic goods implies a 20 percent flexible-price markup. The production subsidy offsets that markup in the reference allocation. The Rotemberg coefficient is 71.464, matching the local Phillips-curve slope associated with a Calvo non-adjustment probability of 0.75. The reference foreign interest rate is tied to the saver discount factor, and a small portfolio cost makes foreign assets stationary without introducing a separate spread shock.

The trade calibration sets non-oil exports to 12 percent of GDP and net crude exports to approximately 3.77 percent. Together with direct household oil expenditure of 1.2 percent of consumption and the production-input mapping, these targets determine the remaining expenditure weights and the oil endowment. The resulting non-oil import weight is 0.15765 and the gross-output oil-input share is 0.01158. The export-demand elasticity is set to two. An aggregate total-goods export projection is included in the companion empirical files, but is not used to replace this elasticity: its petroleum content, destinations, and price index differ from the model’s non-oil export demand after tariffs.

The reference assigns resident oil receipts to savers and sets resident ownership to one. This is a transparent income-incidence assumption for a fixed, costless endowment. It allows expenditure and ownership exposure to be studied separately. The production block describes non-oil activity; it does not convert gross oil receipts into an empirical measure of households’ net petroleum profits. An alternative that sends half of the oil income abroad appears among the mechanism diagnostics.

The mortgage law uses $(\rho, \kappa_m) = (0.3268, 0.5000)$ from the quarterly projection. Only its dynamics are transferred into the model: the empirical intercept is not interpreted as a structural steady-state bank markup. The reference sets the imported-cost coefficient γ_f to zero and evaluates 0.05 separately. The supply–use alternative replaces the energy-expenditure mappings while retaining the rest of the model’s architecture. These alternatives allow measured expenditure magnitudes and credit dynamics to affect the results without changing the underlying policy question.

B Shocks and Policy Comparisons

The experiment starts with foreign and Canadian tariffs at zero. The initial foreign tariff is 5 percent and the Canadian composite counter-tariff is 1.5 percent. Their innovations in the log-wedge representation are $\log(1.05)$ and $\log(1.015)$. The oil-price innovation is $\log(1.20)$. Both driving processes have persistence 0.95. The magnitudes describe effective shocks on the baskets represented in the model; they are not standard-deviation normalizations or a reconstruction of every tariff line. The central experiment features a higher oil price, so its income and expenditure effects are read in that direction.

Four scenarios are solved: no shock, tariffs alone, oil alone, and both shocks together. Under each scenario, the policy alternatives are the CPI rule, domestic price stability, the Ramsey allocation, and the optimized CPI rule. The CPI rule has smoothing coefficient 0.8 and inflation coefficient 1.5. The optimized rule varies these same two coefficients; it does not add responses to output, credit, or the observed shocks. The coefficients are chosen by a numerical search to maximize the joint-episode commitment criterion over $\rho_R \in [0, 0.98]$ and $\phi_\pi \in [1.01, 8]$, subject to determinacy and path admissibility.

All policies inherit the same physical Ramsey reference state and the same planner promises. A rule that converges to a different stationary allocation is therefore evaluated along its transition from that common state. Shock effects are measured relative to the policy’s own no-shock transition, while direct welfare comparisons use the levels of value functions under the same realized shocks. This construction prevents differences in initial wealth or in stationary reference utility from being mistaken for stabilization gains. The commitment criterion and unaugmented utility are reported separately.

C Measurement and Solution

Real GDP measures value added at fixed reference prices. A second measure values the same flows in units of the consumption basket. Their divergence records the role of relative prices; neither is treated as an exact replication of an official chained national-account index. Consumption, debt, and house-price responses are percentage deviations from the relevant no-shock path. Interest rates are annual effective rates: a gross quarterly rate R_t is reported as $100(R_t^4 - 1)$. Inflation is annualized with the same compounding convention. Consumption-equivalent welfare changes are percentages of a permanent consumption adjustment, not annual rate changes.

The main results are nonlinear deterministic paths following the initial surprise. The equilibrium and planner first-order conditions are solved jointly, retaining a path only when the private collateral multiplier and the domains of economic variables are valid. Long solution horizons absorb the slow return of financial states; the main calculation uses 6,000 quarters and explicitly includes continuation values. The figures focus on the economically immediate adjustment. First- and second-order approximations, horizon comparisons, and feasible local policy deviations provide complementary accuracy checks. The branch selection and admissibility conditions are documented in Appendix D.

The nine-configuration sensitivity exercise combines the central calibration, four joint covariance directions for mortgage transmission, two alternative mortgage specifications, the supply–use energy mapping, and positive imported-cost exposure. Policies share the same initial state within each calibration. The exercise evaluates targeted uncertainty about the margins used to interpret the policy response; its selected points are not a probability distribution over optimal policies.

V Policy Responses, Distribution, and Welfare

The policy comparison delivers two distinct findings. Accommodating the joint shock substantially reduces the initial contraction and protects borrowers relative to the CPI rule. Yet the aggregate consumption-equivalent gain is small, and savers prefer the CPI rule to Ramsey. The distinction matters: large differences in impact responses need not imply large permanent welfare gains, and support for indebted households need not be a Pareto improvement. Moreover, the sign of the Ramsey response survives when borrowers become negligible. Household debt changes the allocation and the strength of the policy response; it does not, by itself, explain why accommodation is desirable.

All nonlinear comparisons begin from the same physical Ramsey steady state and evaluate the same inherited promises. Each shock response is measured relative to the corresponding policy’s no-shock transition from that state. The figures therefore remove the transition associated with changing policy, whereas the welfare comparisons use utility levels at the common initial state. The experiment combines a 5 percent US

tariff increase, a 1.5 percent Canadian counter-tariff increase, and a 20 percent increase in the foreign oil price. The two driving processes have quarterly persistence of 0.95. These are conditional experiments in the calibrated economy, with no further surprises after the initial innovations.

A The Source of Inflation and the Policy Trade-off

Figure 3 separates the tariff episode, the oil episode, and their joint realization. The tariff episode produces a particularly sharp divergence between consumer and domestic producer prices. Weaker foreign demand depresses domestic production, while the counter-tariff and the exchange-rate adjustment raise the consumer price index. Under the CPI rule, CPI inflation rises by 0.884 annualized percentage point on impact even as domestic producer inflation falls by 0.708 percentage point. The policy rate rises by 26.71 annual basis points, and real GDP falls by 0.928 percent. Ramsey instead reduces the rate by 11.99 basis points, permits a larger CPI increase, and limits the GDP decline to 0.389 percent. Responding to the headline inflation increase therefore reinforces the contraction associated with lost export demand in this episode.

Oil introduces a different combination of costs and income. Its price raises household expenditure and firms' input costs, but it also increases the income generated by the fixed domestic oil endowment. In the benchmark, residents own all oil rents, which accrue directly to savers. Under the CPI rule, savers' consumption rises by 0.058 percent on impact, whereas borrowers' consumption falls by 0.260 percent. The real exchange rate appreciates by 1.187 percent, dampening the increase in the CPI-deflated oil price. Ramsey also eases in the oil-only episode, but by just 2.31 basis points, compared with 11.99 for tariffs. The initial response to the joint shock is therefore primarily associated with the tariff episode rather than with a generic policy prescription for higher oil prices.

Table 3 shows the combined experiment. The CPI rule raises the policy rate by 33.27 basis points; domestic price stability lowers it by 1.86 basis points; Ramsey lowers it by 13.91 basis points. Real GDP contracts by 1.028, 0.588, and 0.493 percent, respectively. Ramsey approximately halves the initial output decline relative to the CPI rule. It accepts 2.018 percentage points of additional annualized CPI inflation, compared with 1.102 under the CPI rule, while domestic producer inflation rises by only 0.118 percentage point. The relevant stabilization problem is consequently not summarized by the headline CPI response. Much of that response reflects relative-price adjustment across goods, while domestic inflation governs the model's nominal adjustment costs.

Domestic price stability captures an important part of the gain from allowing this adjustment, but it does not reproduce Ramsey. It eliminates domestic price-adjustment costs while forgoing the small domestic inflation movement chosen under commitment. Financial positions, housing allocations, and the distribution of income remain relevant even when producer inflation is zero. The closeness of the output responses under domestic price stability and Ramsey is therefore informative about the importance of domestic price stabilization, not evidence that their allocations or welfare consequences coincide.

B Borrowers Benefit, but the Welfare Gain Is Uneven

Figure 4 traces the joint-shock allocation across the four policy regimes. Under the CPI rule, borrowers' consumption initially falls by 1.128 percent, compared with 0.247 percent for savers. Under Ramsey, the corresponding changes are small increases of 0.062 and 0.013 percent. House prices fall by 0.308 percent

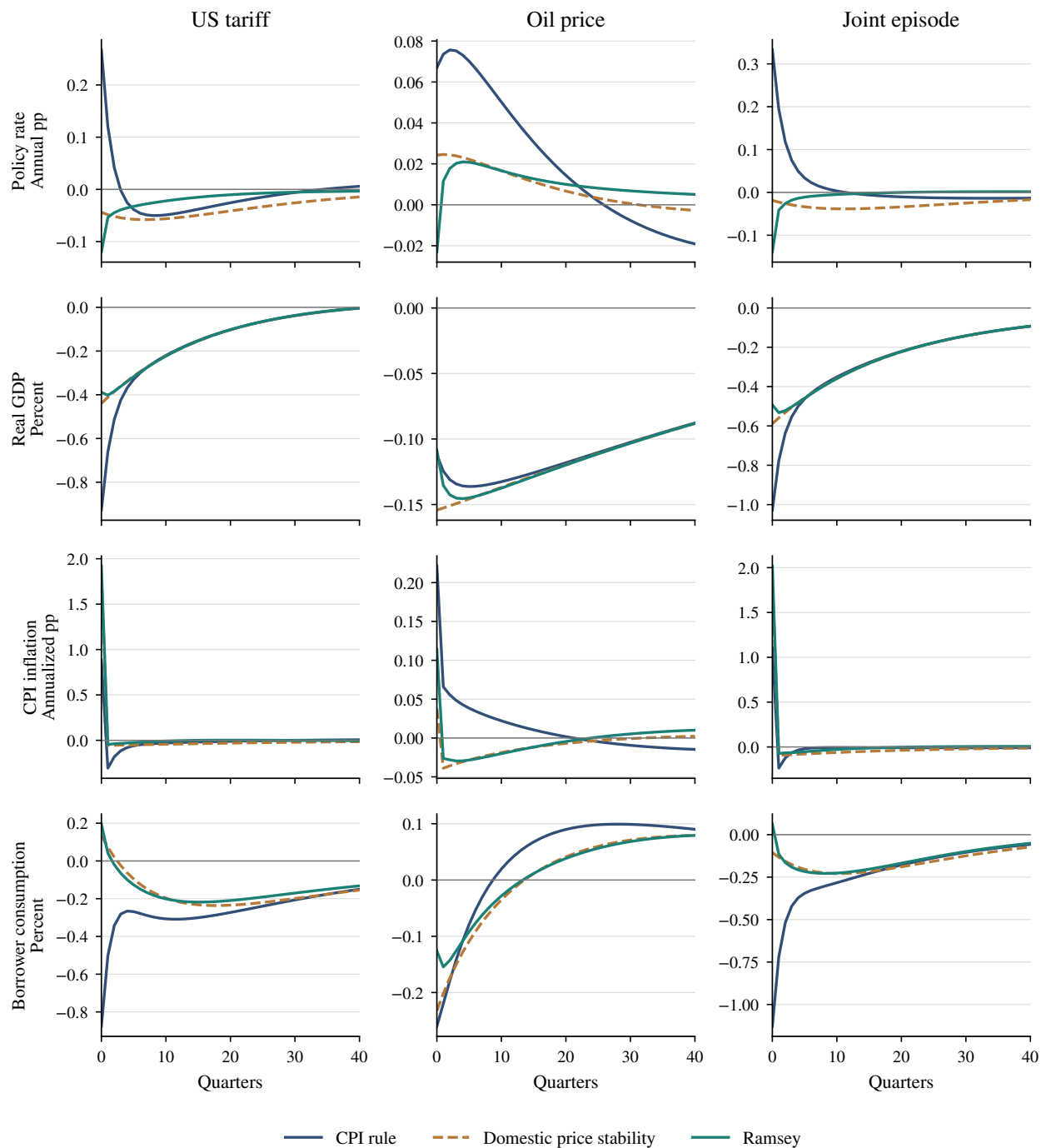


Figure 3. Monetary Policy after Tariff and Oil-Price Shocks

Notes: Columns report the tariff episode, the oil-price episode, and their joint realization. Tariffs rise initially to 5 percent abroad and 1.5 percent in Canada; the foreign oil price rises by 20 percent. Both driving processes have persistence 0.95. Each response is relative to the policy's own no-shock path from the common initial state. Interest rates and inflation are in annual percentage points; GDP and consumption are in percent. Period zero is the impact quarter. The first 40 quarters of the nonlinear paths are shown. Source: calibrated model.

Table 3. The Joint Tariff and Oil Episode: Impact Responses

	CPI rule	Domestic price stability	Ramsey	Optimized CPI rule
Policy rate (annual pp)	0.333	−0.019	−0.139	0.961
Mortgage rate (annual pp)	0.054	−0.003	−0.023	0.156
Real GDP (percent)	−1.028	−0.588	−0.493	−1.185
Value added / CPI (percent)	−0.422	−0.014	0.073	−0.571
Borrower consumption (percent)	−1.128	−0.105	0.062	−1.610
Saver consumption (percent)	−0.247	−0.043	0.013	−0.297
CPI inflation (annualized pp)	1.102	1.848	2.018	1.171
Domestic inflation (annualized pp)	−0.558	0.000	0.118	−0.452
Real house price (percent)	−0.308	−0.096	−0.001	−0.365
Real debt per borrower (percent)	−0.707	1.020	1.673	−0.824

Notes: The foreign tariff is 5 percent, the Canadian composite tariff is 1.5 percent, and the oil price rises by 20 percent. Both shocks have quarterly persistence 0.95. Responses are relative to each policy's no-shock path from the same initial physical state and inherited commitments. Annual percentage-point responses use exact compounding. The optimized CPI rule is selected for this joint episode.

under the CPI rule and are almost unchanged under Ramsey. Borrowers' real debt falls by 0.707 percent in the former case and increases by 1.673 percent in the latter. Because the borrowing constraint remains binding, the latter increase accompanies a reallocation of housing toward borrowers; it should not be read as evidence that their financing constraint has disappeared.

Several channels support this allocation. A smaller decline in labor income helps borrowers, higher unexpected CPI inflation reduces the real service of inherited nominal debt, and future mortgage payments respond to the policy-rate path. These channels interact with collateral and housing demand. Their timing is important: payments at impact depend on the inherited mortgage rate, not the newly chosen rate. The initial consumption response cannot therefore be attributed entirely to an immediate reduction in interest payments. This interaction between income exposure and nominal balance sheets is the relevant distributional margin, consistent with the distinction between aggregate and redistributive monetary transmission in Auclert (2019).

Initial protection also does not imply that borrowers gain from the episode itself. Their Ramsey consumption response becomes negative in the following quarter and reaches about −0.23 percent around two years after the shock. The joint episode reduces their lifetime utility by an amount equivalent to 0.00685 percent of permanent consumption, relative to the Ramsey no-shock path. This loss is much smaller than the 0.07308 percent loss under the CPI rule. Savers, in contrast, gain from the joint episode under both policies, reflecting the oil-income component as well as the subsequent allocation of consumption, housing, and work. The oil-only episode raises unaugmented social welfare in the benchmark, whereas the tariff-only episode lowers it. The positive social consumption equivalent (CE) of the combined episode is thus conditional on an economy that receives substantial oil rents.

Table 4 distinguishes this episode comparison from the policy comparison. At the common initial state, replacing the CPI rule with Ramsey yields a borrower gain of 0.11223 percent of permanent consumption and a saver loss of 0.01766 percent. The unaugmented social CE is only 0.002207 percent. These CEs compensate for changes in total utility, including housing services and labor disutility; they are not averages of consumption responses. Social welfare also uses the planner's discount factor, whereas each household's CE uses its own. The social gain cannot be recovered by taking a population-weighted average of the two

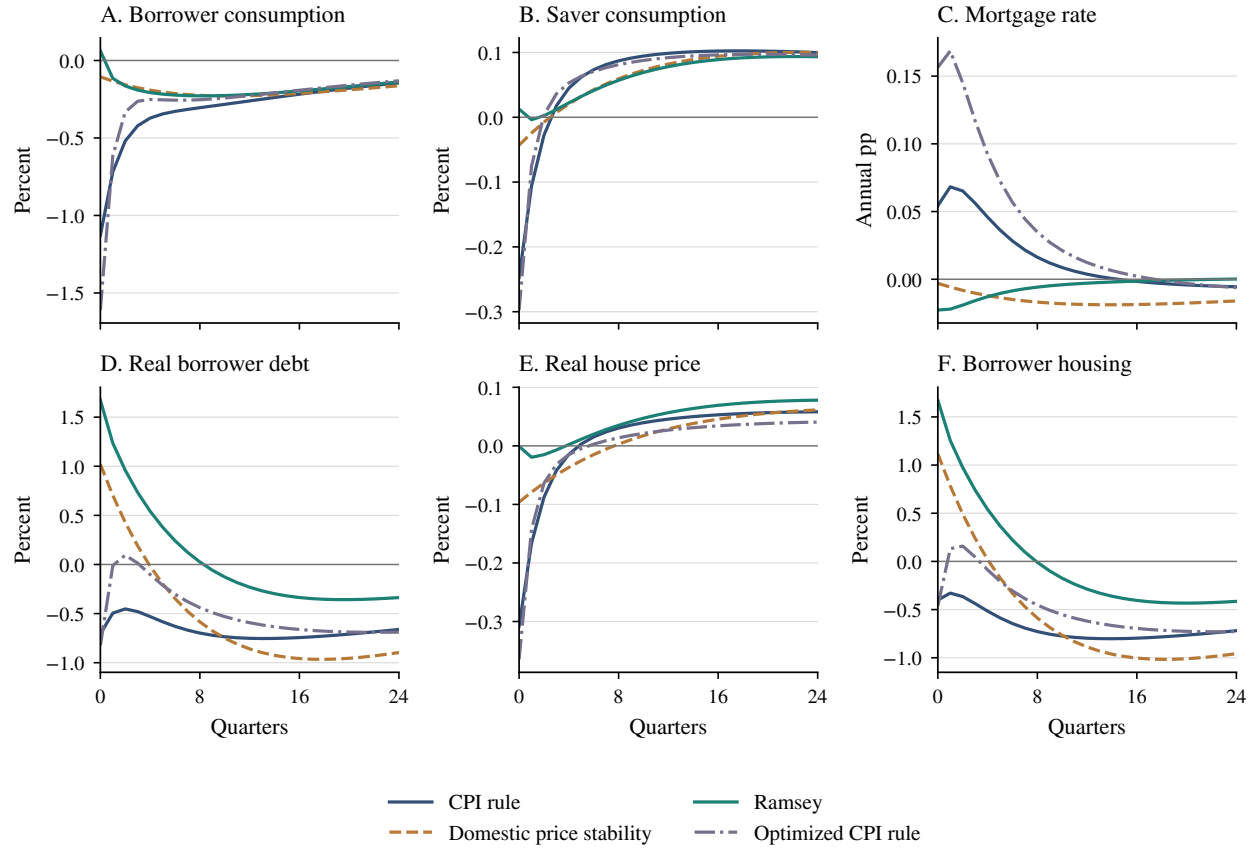


Figure 4. Household Adjustment to the Joint Shock

Notes: Responses to the joint tariff and oil-price episode under four policy regimes. Each path is measured against the corresponding no-shock transition from the common physical state. Consumption, real debt per borrower, the real house price, and housing per borrower are in percent; the mortgage rate is in annual percentage points. The optimized CPI rule is selected for the joint episode using the commitment criterion J . The first 24 quarters are shown. Source: calibrated model.

Table 4. Commitment Values and the Distribution of Welfare Gains

Policy	Consumption equivalent (percent)			Gain in J (10^{-6})	Gap closed (percent)
	Savers	Borrowers	Social W		
CPI rule	0.000000	0.000000	0.000000	0.00	0.00
Domestic price stability	-0.004182	0.042015	-0.000023	211.37	55.49
Ramsey	-0.017658	0.112233	0.002207	380.92	100.00
Optimized CPI rule	-0.000817	0.010707	-0.000099	22.54	5.92

Notes: Every entry compares policies under the same joint shock and initial conditions. Individual consumption equivalents use each household's discount factor; social W uses the planner's discount factor and fixed Pareto weights. The commitment criterion J also includes inherited promises and is not converted into consumption. Gap closed is $(J_p - J_{CPI}) / (J_{Ramsey} - J_{CPI})$, as in equation (19).

household CEs.

Commitment is ranked using the augmented timeless criterion J , which includes the value of inherited promises, rather than unaugmented utility W . In the joint episode, Ramsey improves W over the CPI rule by 0.008827 utility unit but changes the inherited-promise contribution by -0.008446. The net gain in J is therefore 0.000381. Domestic price stability increases J despite a slightly negative unaugmented social

CE. These are different answers to different welfare questions, not conflicting rankings of the same object. Keeping them separate is essential when the economy inherits a commitment history.

C What Can an Optimized CPI Rule Achieve?

I next retain the CPI rule and choose its smoothing and inflation coefficients to maximize J conditional on the joint shock. The best rule found has $(\rho_R, \phi_\pi) = (0.356, 1.262)$, compared with $(0.8, 1.5)$ in the benchmark. It is an interior solution in the searched domain. Its smaller long-run inflation coefficient does not imply a smaller initial response: less smoothing raises the contemporaneous coefficient $(1 - \rho_R)\phi_\pi$ from 0.300 to approximately 0.813.

The optimized rule raises the rate by 96.06 basis points on impact. GDP falls by 1.185 percent and borrower consumption by 1.610 percent, both larger declines than under the benchmark CPI rule. Subsequent borrower consumption recovers more quickly: in the next quarter its response is -0.615 percent, compared with -0.719 under the benchmark. The rule chooses a different intertemporal allocation, not a uniformly milder contraction. Borrowers' total utility is slightly higher than under the benchmark, with a direct CE gain of 0.01071 percent, while savers lose 0.000817 percent.

To assess how much of the Ramsey improvement a simpler policy captures, define

$$\mathcal{G}_p = \frac{J_p - J_{\text{CPI}}}{J_{\text{Ramsey}} - J_{\text{CPI}}}. \quad (19)$$

The optimized CPI rule closes only 5.92 percent of this gap; domestic price stability closes 55.49 percent. These percentages describe the augmented criterion, not fractions of consumption-equivalent gains. Indeed, the optimized rule lowers W by 0.000396 utility unit, more than offset in J by a 0.000419 increase in the inherited-promise contribution. Its direct social CE is -0.0000991 percent. The improvement is small and specifically tied to the commitment criterion.

Changing the coefficients of a CPI-only rule consequently leaves most of the available improvement unexploited. The contrast with domestic price stability suggests that the inflation measure in the rule is economically important in this experiment. It does not establish that the selected coefficients are desirable across environments: they are chosen for this joint realization, and they deliver a lower J than the benchmark CPI rule in the oil-only episode. A rule chosen before an uncertain sequence of shocks is a different policy problem.

D Debt Contracts and the Incidence of Adjustment

The local counterfactuals in Appendix Figure D.1 probe specific transmission margins. They use first-order responses around each economy's own stationary allocation and are interpreted as mechanism comparisons, not welfare rankings across recalibrated economies. Their reference Ramsey rate cut is 14.84 basis points, close to but distinct from the 13.91-basis-point nonlinear common-state response.

In the near-representative-household counterfactual, the borrower share and aggregate debt approach zero together. Ramsey still eases, by 31.22 basis points at impact. The case for accommodation thus persists when borrowers have negligible aggregate weight. The trade and relative-price adjustment already creates a stabilization problem; debt changes its quantitative solution and the incidence of policy. This counterfactual

does not support the stronger claim that indebtedness generates the sign of the optimal interest-rate response.

Holding fixed the price used to value collateral, while retaining borrowers' housing choices, modestly reduces the consumption decline under the CPI rule from 1.177 to 1.144 percent. It also changes the Ramsey response, whose rate cut becomes 17.79 basis points. Removing surprise inflation from loan repayments in the indexed-debt experiment under the CPI rule instead deepens the borrower consumption decline to 1.324 percent. Together these exercises distinguish the collateral-price channel from the revaluation of nominal obligations. They also show why an expansion in borrower debt under Ramsey cannot be assigned to rising house prices alone: in the central nonlinear path, house prices barely change while borrower housing increases.

The oil-income and persistence experiments qualify the distributional result more sharply. When only half of oil rents accrue to residents, borrower consumption falls by 0.072 percent under Ramsey despite an initial rate cut close to the reference. With oil persistence reduced to 0.8, the rate cut shrinks to 7.84 basis points and borrower consumption falls by 0.183 percent. Accommodation therefore does not guarantee a positive initial borrower consumption response. The ownership and duration of the income windfall matter alongside debt contracts.

E Credit Moments Change Magnitudes More Than Signs

The central mortgage mapping separates adjustment speed, $\rho = 0.3268$, from long-run amplitude, $\kappa_m = 0.5000$. Relative to the slower, unit-amplitude reference $(0.05, 1)$, the impact transmission coefficient $\rho \kappa_m$ rises from 0.05 to approximately 0.1634, even though long-run pass-through falls. Ramsey's initial policy-rate cut consequently declines from 25.73 to 13.91 basis points, while its mortgage-rate cut increases from 1.29 to 2.27 basis points. A smaller instrument movement can therefore produce a larger immediate movement in the borrowing rate. The joint change in speed and amplitude, rather than either parameter in isolation, is what the empirical calibration contributes to the policy analysis.

Table 5 reports nine configurations. Across the central estimate and four principal-axis points of its joint HAC covariance ellipse, the Ramsey rate cut ranges from 10.30 to 19.25 basis points. Borrower consumption initially rises by 0.024 to 0.123 percent, and Ramsey ranks above the CPI rule and domestic price stability in J throughout. These selected points describe sensitivity to the estimated reduced-form mapping; their envelope is not a confidence band for optimal policy. Using the mapping under alternative monetary regimes also treats its coefficients as policy invariant, an assumption beyond what the descriptive rate regression establishes.

The other configurations separate credit uncertainty from energy accounting and foreign import costs. Substituting the supply–use energy shares leaves the Ramsey rate cut close to the benchmark, at 15.14 basis points, but changes borrower consumption to -0.065 percent. Introducing $v_{f,t} = \exp(0.05x_t^o)$ changes it to -0.082 percent while leaving Ramsey's rate and real-GDP paths unchanged. The latter invariance, established in Proposition 1 in Appendix C, follows from the model's logarithmic preferences and Cobb–Douglas expenditure system: the specific foreign import-price increase permits a rescaling of CPI-deflated variables that leaves the Ramsey allocation of domestic production unchanged. It is not an empirical finding that import costs are irrelevant to policy; in particular, a CPI feedback rule does not share this invariance.

The durable result across these exercises is the direction of the Ramsey response and its substantial reduction of the initial contraction relative to the CPI rule. The sign of borrowers' impact consumption, the

Table 5. Credit Transmission, Energy Exposure, and the Ramsey Response

Calibration	ρ	κ_m	Policy rate	Real GDP	Borrower C
Central calibration	0.327	0.500	−0.139	−0.493	0.062
Covariance direction 1, −	0.357	0.574	−0.157	−0.486	0.075
Covariance direction 1, +	0.296	0.426	−0.122	−0.499	0.048
Covariance direction 2, −	0.536	0.414	−0.103	−0.507	0.024
Covariance direction 2, +	0.117	0.586	−0.193	−0.473	0.123
Slow, unit-amplitude repricing	0.050	1.000	−0.257	−0.450	0.193
Restricted unit-amplitude projection	0.098	1.000	−0.284	−0.439	0.208
Supply–use energy calibration	0.327	0.500	−0.151	−0.498	−0.065
Imported cost, $\gamma_f = 0.05$	0.327	0.500	−0.139	−0.493	−0.082

Notes: Ramsey impact responses to the joint episode. The policy rate is in annual percentage points; GDP and consumption are in percent. Each calibration uses its own reference equilibrium, with common initial conditions across policies within that calibration. The four covariance directions are selected points on the joint ARX covariance ellipse, not confidence intervals for policy responses. The energy and import-cost alternatives are economic scenarios.

size of the permanent social gain, and the amount of easing are more conditional. This distinction places the economic role of debt precisely: it shapes who absorbs the joint shock and how strongly policy responds, within a stabilization problem that already exists because tariffs and oil move domestic activity, relative prices, and national income differently.

VI Conclusion

Foreign tariffs and oil prices can raise the consumer price index while creating different pressures on domestic production and household resources. In a commodity exporter with collateralized nominal debt, the policy response also determines how these pressures pass through existing loan contracts and household balance sheets. The aggregate terms-of-trade position does not settle that distributional problem.

In the Canadian calibration studied here, the Ramsey allocation accommodates the initial joint shock. Relative to the CPI rule, it substantially reduces the contraction in real GDP and supports borrower consumption, while accepting a larger increase in CPI inflation. The aggregate consumption-equivalent gain is small, and its distribution is uneven: borrowers gain relative to the CPI rule while savers lose. A common initial-state comparison makes these differences explicit without attributing changes in stationary wealth to stabilization policy.

Mortgage transmission is quantitatively important for the size of the required rate movement. Separating the speed of repricing from its amplitude changes the relation between the policy rate and the rate paid on outstanding debt. The sign of accommodation survives across the selected calibrations, but the sign of borrowers' initial consumption response depends on energy expenditure and foreign import costs. The presence of debt therefore calls for an account of incidence and transmission rather than an unconditional policy-rate prescription.

The analytical separability result gives a complementary lesson. Under logarithmic utility and Cobb–Douglas consumption, a specific foreign import-cost increase changes purchasing power without changing the Ramsey nominal rate path. Its policy relevance depends on the margin through which it enters the economy, rather than on its contribution to measured inflation alone. Together with the rule comparisons, this result

identifies which features of external exposure change the domestic stabilization problem and which affect purchasing power while preserving the Ramsey interest-rate path.

Data and replication. The companion replication package contains the archived public data, empirical transformations, model equations, calibration files, nonlinear paths, and validation scripts. The manuscript's source package includes the exact numerical inputs and scripts used to generate its tables and figures.

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Appendix

When Tariffs Meet Oil

A Equilibrium Conditions and Model Details

This appendix records the equilibrium implemented in the replication files. Quantities are per household of the relevant type unless an aggregate is explicitly defined. Domestic values are expressed in units of the consumption price index. The foreign numeraire is fixed at $P_t^* = 1$, and foreign assets are denominated in that numeraire. Both domestic debt and foreign assets have one-quarter maturity.

A.1 Preferences, Demand, and Derived Quantities

For $i \in \{S, B\}$, period utility is

$$u_i(C_i, H_i, L_i) = u_\sigma(C_i) + \xi_i \log H_i - \chi_i \frac{L_i^{1+\psi}}{1+\psi}, \quad u_\sigma(C) = \begin{cases} C^{1-1/\sigma}/(1-1/\sigma), & \sigma \neq 1, \\ \log C, & \sigma = 1. \end{cases} \quad (\text{A.1})$$

Marginal utility is $\Lambda_{i,t} = C_{i,t}^{-1/\sigma}$. The CES consumption aggregator and its Cobb–Douglas limit are

$$C_{i,t} = \left[\sum_{j \in \{h, f, o\}} \omega_j^{1/\eta} C_{ij,t}^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)}, \quad C_{i,t}|_{\eta=1} = \prod_j \left(\frac{C_{ij,t}}{\omega_j} \right)^{\omega_j}. \quad (\text{A.2})$$

Its price index is $P_{C,t} = [\sum_j \omega_j P_{j,t}^{1-\eta}]^{1/(1-\eta)}$, with $P_{C,t} = \prod_j P_{j,t}^{\omega_j}$ at $\eta = 1$. Thus $C_{ij,t} = \omega_j P_{j,t}^{-\eta} C_{i,t}$, and aggregation gives

$$\begin{aligned} C_t &= (1-\lambda)C_{S,t} + \lambda C_{B,t}, & L_t &= (1-\lambda)L_{S,t} + \lambda L_{B,t}, \\ C_{j,t} &= \omega_j P_{j,t}^{-\eta} C_t, & a_t &= \frac{\lambda b_t}{1-\lambda}. \end{aligned} \quad (\text{A.3})$$

All three expenditure components are included in the CPI. The imported composite f is separate from direct oil consumption and productive oil inputs. Its foreign price can nevertheless embody higher energy costs abroad.

External states, prices, and the Canadian tariff rebate are

$$\begin{aligned} x_t^k &= \rho_k x_{t-1}^k + \varepsilon_t^k, & k &\in \{\tau, o\}, & P_{o,t}^* &= \bar{P}_o^* e^{x_t^o}, \\ T_t^{CA} &= e^{\chi_{CA} x_t^\tau}, & v_{f,t} &= e^{\gamma_f x_t^o}, \\ p_{o,t} &= s_t P_{o,t}^*, & p_{f,t}^b &= s_t v_{f,t}, \\ p_{f,t} &= T_t^{CA} p_{f,t}^b, & \mathcal{T}_t &= (T_t^{CA} - 1) p_{f,t}^b C_{f,t}. \end{aligned} \quad (\text{A.4})$$

The export-demand schedule, productive oil demand, and portfolio closure are

$$\begin{aligned}
X_t &= \bar{X} [e^{x_t^\tau} p_{h,t} / s_t]^{-\eta_x}, & NX_t^o &= \bar{Q}_o - C_{o,t} - M_t, \\
mc_t &= \frac{1}{A} \left[\frac{w_t}{(1 - \alpha_o) p_{h,t}} \right]^{1 - \alpha_o} \left[\frac{p_{o,t}}{\alpha_o p_{h,t}} \right]^{\alpha_o}, & M_t &= \frac{\alpha_o mc_t p_{h,t} Y_t}{p_{o,t}}, \\
g(f_t) &= \frac{\Psi_F}{2\bar{V}} (f_t - \bar{f})^2, & g'(f_t) &= \frac{\Psi_F}{\bar{V}} (f_t - \bar{f}), \\
F_t &= (1 - \lambda) f_t, & G_t &= (1 - \lambda) g(f_t).
\end{aligned} \tag{A.5}$$

The scale $\bar{V}$ is fixed and positive. For the limit $\alpha_o = 0$, $mc_t = w_t / (A p_{h,t})$ and $M_t = 0$.

A.2 The 17 Private Equilibrium Conditions

The endogenous vector contains 18 variables:

$$\begin{aligned}
y_t &= (C_{S,t}, C_{B,t}, H_{S,t}, H_{B,t}, L_{S,t}, L_{B,t}, b_t, \mu_t, f_t, \\
&\quad p_{h,t}, s_t, \pi_{C,t}, \pi_{h,t}, q_t, w_t, Y_t, R_t^m, R_t).
\end{aligned} \tag{A.6}$$

The policy instrument is included in this vector. The following 17 conditions, the two exogenous processes, and a monetary-policy condition close the decentralized economy. Expectations become realized future values along the deterministic paths reported in the paper.

The saver conditions are

$$0 = \Lambda_{S,t} - \beta_S R_t \mathbb{E}_t \left[\frac{\Lambda_{S,t+1}}{\Pi_{C,t+1}} \right], \tag{E1}$$

$$0 = s_t \Lambda_{S,t} [1 + g'(f_t)] - \beta_S R^* \mathbb{E}_t [s_{t+1} \Lambda_{S,t+1}], \tag{E2}$$

$$0 = q_t \Lambda_{S,t} - \frac{\xi_S}{H_{S,t}} - \beta_S \mathbb{E}_t [\Lambda_{S,t+1} q_{t+1}], \tag{E3}$$

$$0 = w_t \Lambda_{S,t} - \chi_S L_{S,t}^\Psi. \tag{E4}$$

Equation (E2) prices a foreign-currency payoff, converted by s_{t+1} . A second division by domestic CPI inflation would therefore be incorrect. The portfolio cost is paid when the asset is acquired, which accounts for $1 + g'(f_t)$ on the left-hand side.

The borrower conditions in the binding-collateral region are

$$0 = \Lambda_{B,t} - \beta_B R_t^m \mathbb{E}_t \left[\frac{\Lambda_{B,t+1}}{\Pi_{C,t+1}} \right] - \mu_t, \tag{E5}$$

$$0 = q_t \Lambda_{B,t} - m q_t \mu_t - \frac{\xi_B}{H_{B,t}} - \beta_B \mathbb{E}_t [\Lambda_{B,t+1} q_{t+1}], \tag{E6}$$

$$0 = w_t \Lambda_{B,t} - \chi_B L_{B,t}^\Psi, \tag{E7}$$

$$0 = C_{B,t} + q_t (H_{B,t} - H_{B,t-1}) + \frac{R_{t-1}^m b_{t-1}}{\Pi_{C,t}} - w_t L_{B,t} - b_t - \mathcal{T}_t, \tag{E8}$$

$$0 = b_t - m q_t H_{B,t}. \tag{E9}$$

Admissibility additionally requires $\mu_t \geq 0$, positive consumption, housing, hours and relevant prices, and positive gross inflation rates. The borrowing limit is imposed on principal at current house value; there is no future house price or interest-rate factor in (E9).

Housing clearing and price identities are

$$0 = (1 - \lambda)H_{S,t} + \lambda H_{B,t} - \bar{H}, \quad (\text{E10})$$

$$0 = \mathcal{P}_t, \quad \mathcal{P}_t = \begin{cases} \sum_j \omega_j \log p_{j,t}, & \eta = 1, \\ \sum_j \omega_j p_{j,t}^{1-\eta} - 1, & \eta \neq 1, \end{cases} \quad (\text{E11})$$

$$0 = \Pi_{h,t} - \frac{p_{h,t}}{p_{h,t-1}} \Pi_{C,t}. \quad (\text{E12})$$

The final five conditions are

$$0 = w_t L_t - (1 - \alpha_o) m c_t p_{h,t} Y_t, \quad (\text{E13})$$

$$0 = \varphi \Pi_{h,t} \pi_{h,t} - (1 - \varepsilon)(1 + \varsigma) - \varepsilon m c_t - \frac{\varepsilon \varphi}{2} \pi_{h,t}^2 - \beta_S \mathbb{E}_t \left[\frac{\Lambda_{S,t+1}}{\Lambda_{S,t}} \frac{\Pi_{h,t+1}}{\Pi_{C,t+1}} \frac{Y_{t+1}}{Y_t} \varphi \Pi_{h,t+1} \pi_{h,t+1} \right], \quad (\text{E14})$$

$$0 = Y_t - C_{h,t} - X_t - \mathcal{D}_t, \quad \mathcal{D}_t = \frac{\varphi}{2} \pi_{h,t}^2 Y_t, \quad (\text{E15})$$

$$0 = s_t(F_t + G_t) - s_t R^* F_{t-1} - p_{h,t} X_t - p_{o,t} N X_t^o + p_{f,t}^b C_{f,t} + (1 - \kappa_o) p_{o,t} \bar{Q}_o, \quad (\text{E16})$$

$$0 = (R_t^m - \bar{R}) - (1 - \rho)(R_{t-1}^m - \bar{R}) - \rho \kappa_m (R_t - \bar{R}). \quad (\text{E17})$$

The CPI-rule closure is (16), with an additive innovation ε_t^R when a monetary shock is used for diagnostics. The domestic-price-stability closure is $\pi_{h,t} = 0$. No lower bound on the policy rate is imposed.

A.3 Price Setting and Flow Accounts

Let $P_{h,j,t}$ be a domestic variety's price, with isoelastic demand $Y_{j,t} = (P_{h,j,t}/P_{h,t})^{-\varepsilon} Y_t$. A firm pays

$$\mathcal{D}_{j,t} = \frac{\varphi}{2} \left(\frac{P_{h,j,t}}{P_{h,j,t-1}} - 1 \right)^2 Y_{j,t} \quad (\text{A.7})$$

units of the domestic aggregate, valued at $P_{h,t}$. The firm takes the aggregate price as given and discounts nominal profits using the saver pricing kernel $\beta_S \Lambda_{S,t+1}/(\Lambda_{S,t} \Pi_{C,t+1})$. Adjustment costs are proportional to the firm's own sales. Differentiating that sales term generates $\varepsilon \varphi \pi_{h,t}^2/2$ in the symmetric pricing condition (E14); differentiating next period's price change generates its forward-looking term. This specification, rather than an adjustment cost scaled by exogenous aggregate sales, determines the nonlinear equation used here.

In consumption units, aggregate operating profits and transfers are

$$\begin{aligned}\mathcal{P}_{h,t} &= (1 + \varsigma)p_{h,t}Y_t - w_tL_t - p_{o,t}M_t - p_{h,t}\mathcal{D}_t, \\ \mathcal{P}_{o,t} &= \kappa_o p_{o,t}\bar{Q}_o, \\ T_{S,t} &= -\frac{\varsigma p_{h,t}Y_t}{1 - \lambda}.\end{aligned}\tag{A.8}$$

$$\mathcal{P}_{b,t} = \lambda \frac{(R_{t-1}^m - R_{t-1})b_{t-1}}{\Pi_{C,t}},$$

All domestic profits and resident oil receipts accrue to savers. The subsidy tax T_S is separate from the per-capita tariff rebate $\mathcal{T}_t$. The saver budget is

$$\begin{aligned}C_{S,t} + q_t(H_{S,t} - H_{S,t-1}) + a_t + s_t[f_t + g(f_t)] \\ = w_tL_{S,t} + T_{S,t} + \frac{R_{t-1}a_{t-1}}{\Pi_{C,t}} + s_tR^*f_{t-1} \\ + \frac{\mathcal{P}_{h,t} + \mathcal{P}_{o,t} + \mathcal{P}_{b,t}}{1 - \lambda} + \mathcal{T}_t.\end{aligned}\tag{A.9}$$

After aggregation, domestic loan and deposit positions cancel, as do the subsidy and its financing. Housing trades cancel by (E10). With goods demand and the balance of payments, (A.9) is implied by the other conditions and serves as the Walras-law check rather than an additional equilibrium restriction. Oil payments to nonresidents remain an external income debit.

The volume measure uses $p_h^{\text{ref}} = 1$ and $p_o^{\text{ref}} = \bar{P}_o^*$. The two GDP definitions and nominal depreciation are

$$\begin{aligned}GDP_t^{CPI} &= p_{h,t}(Y_t - \mathcal{D}_t) - p_{o,t}M_t + p_{o,t}\bar{Q}_o, \\ GDP_t^{\text{vol}} &= Y_t - \mathcal{D}_t - \bar{P}_o^*M_t + \bar{P}_o^*\bar{Q}_o, \\ \frac{E_t}{E_{t-1}} &= \frac{s_t}{s_{t-1}}\Pi_{C,t}.\end{aligned}\tag{A.10}$$

An annual-effective rate in percent is $100(R_t^4 - 1)$; annual-effective inflation is $100(\Pi_{C,t}^4 - 1)$. Differences of these expressions are percentage points per year. For an order-one response of any nonlinear observable, its Jacobian is evaluated at the relevant policy steady state. The nonlinear experiments evaluate the exact definitions along nonlinear equilibrium paths.

A.4 Reference Allocation and Calibrated Preference Parameters

Bars in this subsection denote the private zero-inflation reference allocation. Set $\bar{p}_h = \bar{s} = \bar{P}_o^* = 1$, $\bar{R} = R^* = 1/\beta_S$, $\bar{R}^m = \bar{R}$, and $\varsigma = 1/(\varepsilon - 1)$, so that $\bar{m}\bar{c} = 1$. Let $\bar{D}$ denote the GDP normalization, $n_o = \bar{N}\bar{X}^o/\bar{D}$ the oil net-export target, and $\bar{F}$ the aggregate foreign-asset target. The oil endowment and aggregate consumption are

$$\begin{aligned}\bar{Q}_o &= \frac{(1 - \alpha_o)n_o\bar{D} + \omega_o(1 - \alpha_o)[\bar{D} + (R^* - 1)\bar{F}] + \alpha_o\bar{D}}{1 + \omega_o(1 - \alpha_o)(1 - \kappa_o)}, \\ \bar{C} &= \bar{D} - (1 - \kappa_o)\bar{Q}_o + (R^* - 1)\bar{F}.\end{aligned}\tag{A.11}$$

These expressions impose the oil trade identity and aggregate income account simultaneously. They do not independently target an oil-sector profit series. Production and openness then satisfy

$$\begin{aligned}\bar{Y} &= \frac{\bar{D} - \bar{Q}_o}{1 - \alpha_o}, & \bar{M} &= \alpha_o \bar{Y}, & \bar{w} &= \frac{(1 - \alpha_o) \bar{Y}}{\bar{L}}, \\ A &= \frac{\bar{Y}}{\bar{L}^{1 - \alpha_o} \bar{M}^{\alpha_o}}, & \omega_h &= \frac{\bar{Y} - \bar{X}}{\bar{C}}, & \omega_f &= 1 - \omega_h - \omega_o.\end{aligned}\tag{A.12}$$

The aggregate labor input is $\bar{L} = (1 - \lambda) \bar{L}_S + \lambda \bar{L}_B$ and $\bar{f} = \bar{F} / (1 - \lambda)$. At $\alpha_o = 0$ use $A = \bar{Y} / \bar{L}$. If h_D and b_D denote housing-value and aggregate debt ratios to quarterly GDP, housing allocations follow from

$$\bar{q} = \frac{h_D \bar{D}}{\bar{H}}, \quad \bar{b} = \frac{b_D \bar{D}}{\lambda}, \quad \bar{H}_B = \frac{\bar{b}}{m \bar{q}}, \quad \bar{H}_S = \frac{\bar{H} - \lambda \bar{H}_B}{1 - \lambda}.\tag{A.13}$$

Budgets determine the household consumption levels:

$$\begin{aligned}\bar{C}_B &= \bar{w} \bar{L}_B - (\bar{R}^m - 1) \bar{b}, \\ \bar{C}_S &= \bar{w} \bar{L}_S + (\bar{R} - 1) \bar{a} + (R^* - 1) \bar{f} + \frac{\kappa_o \bar{Q}_o}{1 - \lambda}.\end{aligned}\tag{A.14}$$

Writing $\bar{\Lambda}_i = \bar{C}_i^{-1/\sigma}$, the remaining preference parameters and the multiplier are

$$\begin{aligned}\bar{\mu} &= \bar{\Lambda}_B (1 - \beta_B \bar{R}^m), \\ \xi_S &= \bar{q} \bar{H}_S \bar{\Lambda}_S (1 - \beta_S), \\ \xi_B &= \bar{q} \bar{H}_B \bar{\Lambda}_B [(1 - \beta_B) - m(1 - \beta_B \bar{R}^m)], \\ \chi_i &= \frac{\bar{w} \bar{\Lambda}_i}{\bar{L}_i^\psi}, \quad i \in \{S, B\}.\end{aligned}\tag{A.15}$$

Positive reference consumption, housing and utility weights, and $\bar{\mu} > 0$, are checked after this calibration. The normalized Pareto weights are

$$\alpha_i = \frac{1/\bar{\Lambda}_i}{(1 - \lambda)/\bar{\Lambda}_S + \lambda/\bar{\Lambda}_B}, \quad (1 - \lambda) \alpha_S + \lambda \alpha_B = 1.\tag{A.16}$$

They remain fixed across policy comparisons within a parameterization. The parameter θ used to calibrate price rigidity is a slope-matching device: $\varphi = \varepsilon / \kappa_\pi$, where $\kappa_\pi = (1 - \theta)(1 - \beta_S \theta) / \theta$. The model itself uses the Rotemberg equation (E14).

A.5 Commitment and Welfare Evaluation

Let g_t stack the residuals (E1)–(E17) in that order. For a deterministic path, the discounted Lagrangian is

$$\mathcal{L} = \sum_{t \geq 0} \beta_S^t [U(y_t) + \zeta_t^\top g_t(y_{t-1}, y_t, y_{t+1})] + \beta_S^{-1} \zeta_{-1}^\top g_{-1}(y_{-2}, y_{-1}, y_0).\tag{A.17}$$

The 18 stationarity conditions, one for each element $y_{i,t}$ including R_t , are

$$0 = U_i(y_t) + \sum_{j=1}^{17} \left[\zeta_{j,t} \frac{\partial g_{j,t}}{\partial y_{i,t}} + \beta_S \zeta_{j,t+1} \frac{\partial g_{j,t+1}}{\partial y_{i,t}} + \beta_S^{-1} \zeta_{j,t-1} \frac{\partial g_{j,t-1}}{\partial y_{i,t}} \right]. \quad (\text{A.18})$$

Together with the private constraints they characterize the Ramsey stationary path. The steady-state search solves the private equations for a candidate policy rate, obtains the 17 multipliers from the noninstrument conditions, and imposes the instrument condition. Candidate roots must also satisfy all private and planner residuals and the economic domain. When several candidates pass, the reported branch is the one nearest the private reference rate. These first-order and admissibility conditions do not establish global optimality.

Each policy starts from the same Ramsey physical prehistory. Its own no-shock transition is solved from that state, and its shocked path is compared with that transition. The inherited ζ_{-1} entering (18) is common to the policy comparisons. The utility recursions are

$$V_{i,t} = u_{i,t} + \beta_i V_{i,t+1}, \quad W_t = U_t + \beta_S W_{t+1}. \quad (\text{A.19})$$

With logarithmic consumption utility and the normalization (A.16), common-state utility gains have the exact consumption-equivalent representation stated in the main text. An episode effect subtracts the value of the same policy's no-shock transition; a gain relative to the CPI rule subtracts that rule's value for the same shock and initial state. Neither subtraction changes the definition of the separate augmented criterion J .

A.6 Mechanism Variants

The frozen-collateral-price variant replaces (E9) by $b_t = m\bar{q}H_{B,t}$ and replaces $mq_t\mu_t$ in (E6) by $m\bar{q}\mu_t$. It preserves the private reference allocation and preferences while removing movements in the price used by the borrowing limit. Setting $\rho = 1$ makes the effective borrowing rate adjust immediately to $\bar{R} + \kappa_m(R_t - \bar{R})$; it does not impose unit pass-through when $\kappa_m \neq 1$.

The indexed-debt diagnostic introduces $\pi_t^e = \mathbb{E}_t \pi_{C,t+1}$. Borrower debt service becomes $R_{t-1}^m b_{t-1} / (1 + \pi_{t-1}^e)$, and its Euler condition uses $\beta_B R_t^m \mathbb{E}_t \Lambda_{B,t+1} / (1 + \pi_t^e)$. Deposits remain nominal. Intermediary profits are therefore

$$\mathcal{P}_{b,t}^{\text{ind}} = \lambda b_{t-1} \left[\frac{R_{t-1}^m}{1 + \pi_{t-1}^e} - \frac{R_{t-1}}{\Pi_{C,t}} \right]. \quad (\text{A.20})$$

This change preserves the aggregate accounts by assigning the difference in loan and deposit indexation to the saver-owned intermediaries. The reported indexed-debt exercise is a first-order mechanism diagnostic; the nonlinear policy comparisons use the nominal-debt model.

B Data Construction and Empirical Methods

B.1 Samples and Data Vintages

The empirical calculations use data downloaded in October 2026, with observations truncated at 2025Q4. They are retrospective calculations on revised data, not estimates based on the information set available at the end of 2025. Mortgage observations come directly from the Bank of Canada's public data service.

National-account series use archived DBnomics copies of Statistics Canada tables when the official download endpoints were unavailable. The income and terms-of-trade extract was indexed in September 2026; the supply and use extract was indexed in November 2025. The imported-consumer-goods series is an inactive historical table ending in 2018Q2. No interpolation extends that price series beyond its observed endpoint.

The trade exposure of 75.9 percent refers to 2024 domestic merchandise exports to the United States, measured on a customs basis (Global Affairs Canada, 2025). It includes energy and excludes re-exports. The bilateral energy statistics in Canada Energy Regulator (2025) have their own product definitions. These exposure measures motivate the setting but do not identify the model’s tariff elasticity or reconstruct a product-level tariff schedule. The tariff changes in the counterfactual remain effective wedges on the represented baskets.

B.2 Mortgage Aggregation and Dynamic Projection

The outstanding mortgage-rate series combines insured and uninsured rates, V122667787 and V122667793, using balances V122667736 and V122667742. Rates on advances, V122667775 and V122667781, are weighted by flows V122667724 and V122667730. The policy rate is the target overnight rate, V39079. These series cover banks and foreign bank branches; other lenders are outside the reporting universe (Bank of Canada, 2026a,b). Outstanding observations begin in July 2016. From January 2019, residential mortgages to the corporate sector are excluded. A supplementary post-break estimate begins in 2019Q2 and uses 2019Q1 as its first lag.

Monthly insured and uninsured rates are weighted by the corresponding monthly volumes. The main quarterly specification selects the last month of each quarter and the last applicable policy rate. To express the observations in units consistent with the model, an annual rate i_t , quoted in percent, is transformed as

$$r_t = 10^4 \left[\left(1 + \frac{i_t}{100} \right)^{1/4} - 1 \right]. \quad (\text{B.1})$$

This is an observation convention based on an effective annual rate. It does not recover the heterogeneous contractual compounding conventions, fees, or principal repayments embedded in the underlying loans.

I estimate equation (1) by ordinary least squares. The restricted specification is

$$r_t^m - r_{t-1}^m = c + \rho(r_t - r_{t-1}^m) + u_t, \quad (\text{B.2})$$

which imposes $a + b = 1$. The unrestricted estimates imply $a = 0.6732$ and $b = 0.1634$. Inference uses the Bartlett-kernel covariance estimator of Newey and West (1987), four quarterly lags, the finite-sample scaling $n/(n-k)$, and a t_{n-k} reference distribution. The covariance of (ρ, κ_m) is obtained by the delta method from the full coefficient covariance. Their estimated correlation is -0.656 , so the policy sensitivity exercise uses joint covariance directions rather than independent marginal endpoints.

The fitted autoregressive residual has first-order correlation 0.637. This persistence, the lagged dependent variable, and endogenous policy rates limit a structural interpretation of the projection. The intercept absorbs the mean spread and differences in observed rate levels. Importing the dynamic coefficients into an equation centered on the model’s steady state does not estimate that steady-state spread. The restricted adjustment coefficient is 0.098; estimating directly in published annual rates gives 0.099, while a quarterly-average

variant gives 0.094.

For the 2023Q1–2025Q4 validation, coefficients are estimated through 2022Q4 and then held fixed. All predictions condition on the observed future policy rate. One-step predictions use the observed lagged mortgage rate; recursive predictions use only the initial observation and subsequently recycle predicted mortgage rates. Table 1 reports both criteria. Neither exercise is an unconditional forecast of policy or mortgage rates. The dynamic calibration concerns the interest-rate component of debt payments; it does not estimate amortization or total debt service.

ToTEM III provides a useful institutional comparison (Corrigan et al., 2021). Its rate on household debt combines new lending with renegotiation of existing balances. The weight on the new-loan rate is $L_t/D_t + \Phi(1 - L_t/D_t)$, where Φ is the renegotiated fraction of the existing balance. This object differs from ρ in a projection of the outstanding rate on the policy rate. ToTEM’s calibrated $\Phi = 0.093$ and quarterly amortization coefficient 0.0161 would imply a total updating weight of 0.1076 only under a stationary nominal stock with $L/D = 0.0161$. I do not transfer that arithmetic into the estimated projection. More broadly, ToTEM’s Bayesian estimation uses 50 series, including 40 Canadian and 10 foreign observables; its simulated responses are not treated as observed moments here.

B.3 Oil-News Local Projections

The shock series is the column named *Oil supply news shock* in the December 2025 update of the data accompanying Känzig (2021). It contains recovered structural shocks. The separate oil-supply-surprise column is the high-frequency identifying instrument and is not used as the projection regressor. Monthly shocks are summed within quarters. WTI, US consumer prices, world industrial production, and the exchange rate are averaged within quarters. The exchange rate is Federal Reserve H.10 series EXCAUS, Canadian dollars per US dollar; the oil and macroeconomic controls are supplied with the published VAR data.

For each outcome y and horizon $h \in \{0, 1, 2, 4, 8\}$ quarters, I estimate

$$100(\log y_{t+h} - \log y_{t-1}) = \alpha_h + \beta_h s_t + \sum_{j=1}^2 \delta_{hj} s_{t-j} + \sum_{j=1}^2 \Gamma'_{hj} \Delta z_{t-j} + e_{t+h}, \quad (\text{B.3})$$

where s_t is the standardized quarterly shock and z_t contains log real WTI, the exchange rate, world industrial production, and the US consumer price index. The historical import-price specification also includes the log Canadian-dollar import deflator among these lagged controls. No contemporaneous endogenous variable is included as a control. At each horizon, outcomes share the same estimation sample; the last regressor date is shortened to keep all future observations within the available window. There are 111 impact observations in the 1990Q4–2018Q2 import sample. The exchange-rate extension through 2025Q4 uses 141 impact observations.

Responses are rescaled by the estimated impact response of real oil prices, so that the normalization is ten log points, not a literal ten percent level increase. I retain the cross-equation covariance when applying the delta method to the ratios. HAC bandwidth is $\max(4, h + 1)$, with the same finite-sample scaling and t reference as above. The intervals are pointwise and conditional on the published shock extraction; they do not re-estimate that VAR. The estimated oil response need not follow an AR(1), so these projections do not identify the model’s oil-price persistence.

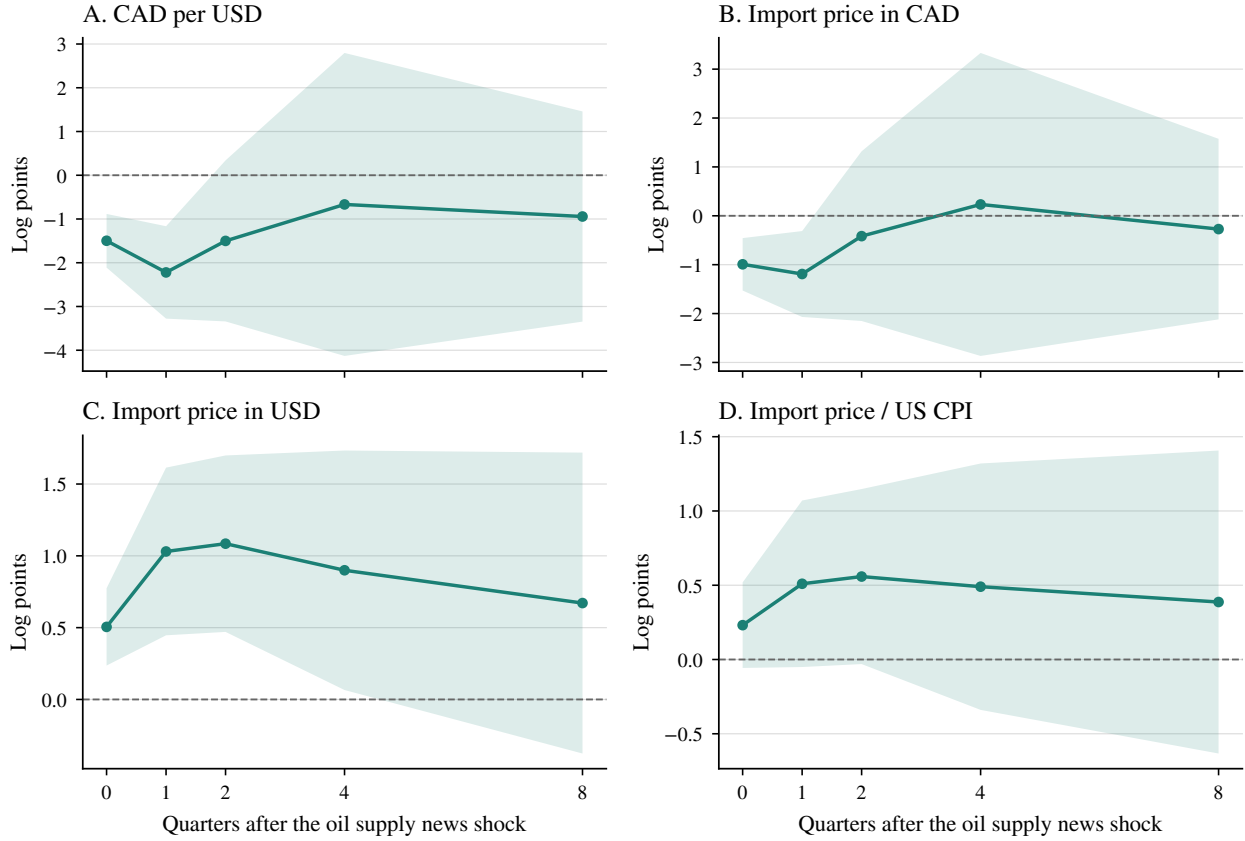


Figure B.1. Oil-Supply News, the Canadian Dollar, and Import Prices

Notes: Quarterly local projections using the structural oil-supply-news shock recovered from the identified VAR of Känzig (2021). Responses are normalized to a ten-log-point increase in real WTI at impact. Import prices refer to the historical consumer-goods deflator. The US-dollar price is the Canadian-dollar price divided by CAD per USD; the foreign relative price additionally divides by US CPI. Shading shows pointwise 95 percent HAC intervals, with uncertainty in the estimated oil-impact normalization incorporated by the delta method. Inference is conditional on the supplied structural-shock series. The effective sample is 1990Q4–2018Q2, with horizon-specific endpoint adjustments. Appendix B.3 specifies the regression and data construction. Sources: Känzig (2021), Statistics Canada (Statistics Canada, 2018), and author’s calculations.

The Canadian-dollar import deflator is 100 times current-price consumer-goods imports divided by their chained-2007-dollar counterpart in table 36-10-0110-01 (Statistics Canada, 2018). Both series are seasonally adjusted at annual rates, so the common annualization cancels. Their coordinates are 1.2.1.91 and 1.1.1.91. Energy is a separate category in the source. I neither add chained volumes across categories nor interpret this partial basket as the complete non-energy import aggregate. Foreign-currency and relative prices are constructed as

$$P_{M,t}^{USD} = P_{M,t}^{CAD} / E_t, \quad P_{M,t}^* = P_{M,t}^{USD} / P_{C,t}^{US}. \quad (\text{B.4})$$

These transformations retain the same observations and controls, ensuring that the estimated log USD-price response equals the CAD-price response minus the exchange-rate response.

B.4 National Accounts and Expenditure Ratios

Real GDP, real gross domestic income, and terms of trade come from table 36-10-0105-01 (Statistics Canada, 2026). Figure 2 rebases the production and income indexes consistently; the historical changes discussed in the text are endpoint ratios of rounded quarterly indexes. They are not identified oil-shock effects or measures of net rents distributed to households.

The expenditure exercise selects refined petroleum products excluding petrochemicals, purchaser prices, household final consumption, and total industries in the national supply and use accounts (Statistics Canada, 2025). The 2017–2019 ratios divide sums of nominal expenditures by sums of nominal GDP or consumption. Multiplying the consumption ratio by 0.380 gives the alternative direct oil weight 0.01424; multiplying industry expenditure relative to GDP by 0.519 gives a crude-input proxy of 0.01738. The conversion factors approximate crude content and remain assumptions. Industry coverage exceeds the modeled non-oil producer, and the accounts have different revision vintages. The resulting alternative is an accounting calibration whose policy consequences can be evaluated, rather than an input-output estimate of the complete model.

C Import-Price Separability

This appendix establishes a property of the model’s logarithmic and Cobb–Douglas specification. It concerns an additional cost in the particular foreign consumption bundle purchased by Canada, holding fixed the oil-price and tariff paths. It does not assert neutrality of the oil shock itself or of a common increase in all foreign prices.

Proposition 1 (Separability of the specific import-price component). *Consider two economies satisfying (E1)–(E17), with nominal debt, current-price collateral $b_t = m q_t H_{B,t}$, and $\eta = \sigma = 1$. All parameters, Pareto weights, exogenous paths, and foreign competitor prices are identical, except that $\gamma_f^1 = \gamma_f^0 + \Delta \gamma_f$. Define*

$$v_t = \frac{v_{f,t}^1}{v_{f,t}^0} = \exp(\Delta \gamma_f x_t^o), \quad \mathcal{A}_t = v_t^{-\omega_f}. \quad (\text{C.1})$$

There is an invertible transformation between their private equilibrium paths that preserves housing, labor, foreign assets, domestic output, both interest rates, and producer inflation, while

$$\begin{aligned} z_t^1 &= \mathcal{A}_t z_t^0, & z &\in \{C_S, C_B, b, p_h, s, q, w\}, \\ \mu_t^1 &= \mathcal{A}_t^{-1} \mu_t^0, & \Pi_{C,t}^1 &= \Pi_{C,t}^0 \frac{\mathcal{A}_{t-1}}{\mathcal{A}_t}. \end{aligned} \quad (\text{C.2})$$

With correspondingly transformed boundary conditions, this map also preserves domestic-price-stability equilibria and Ramsey stationary paths. If optimal commitment allocations exist, it maps the sets of optima into each other. Corresponding paths have identical nominal policy and borrowing rates and identical domestic production, but different composite consumption and import volumes.

Proof. First consider prices and demands. Since $p_{f,t}^{b,1} = \mathcal{A}_t v_t p_{f,t}^{b,0}$, $p_{f,t}^1 = \mathcal{A}_t v_t p_{f,t}^0$ and $p_{o,t}^1 = \mathcal{A}_t p_{o,t}^0$, the transformed log price-index restriction differs from the original by $\log \mathcal{A}_t + \omega_f \log v_t = 0$. Cobb–Douglas

demands give, both by household and in aggregate,

$$C_{h,t}^1 = C_{h,t}^0, \quad C_{o,t}^1 = C_{o,t}^0, \quad C_{f,t}^1 = C_{f,t}^0/v_t. \quad (\text{C.3})$$

The ratios $w_t/p_{h,t}$, $p_{o,t}/p_{h,t}$ and $p_{h,t}/s_t$ are unchanged. Thus marginal cost, productive oil use M_t , exports X_t and aggregate labor are unchanged. So are price-adjustment resources $\mathcal{D}_t$ and GDP at fixed reference prices.

The CPI level changes according to $P_{C,t}^1 = P_{C,t}^0/\mathcal{A}_t$. Its inflation rate therefore has the ratio in (C.2), rather than its inverse. Nominal exchange rates, domestic output prices, wages, house prices and nominal debt $B_t = P_{C,t}b_t$ are unchanged. In particular, inherited debt service satisfies

$$\frac{R_{t-1}^m b_{t-1}^1}{\Pi_{C,t}^1} = \mathcal{A}_t \frac{R_{t-1}^m b_{t-1}^0}{\Pi_{C,t}^0}. \quad (\text{C.4})$$

Borrower budget terms and the collateral limit consequently scale by $\mathcal{A}_t$. Border expenditure satisfies $p_{f,t}^{b,1} C_{f,t}^1 = \mathcal{A}_t p_{f,t}^{b,0} C_{f,t}^0$, as do tariff receipts. Every CPI-denominated balance-of-payments item has the same factor. The saver budget, domestic and oil receipts, bank profits and fiscal transfers scale consistently. Positivity and the sign of the private collateral multiplier are preserved.

For the forward-looking conditions, logarithmic marginal utility implies $\Lambda_{i,t}^1 = \Lambda_{i,t}^0/\mathcal{A}_t$, and hence

$$\frac{\Lambda_{i,t+1}^1}{\Pi_{C,t+1}^1} = \frac{1}{\mathcal{A}_t} \frac{\Lambda_{i,t+1}^0}{\Pi_{C,t+1}^0}, \quad q_t^1 \Lambda_{i,t}^1 = q_t^0 \Lambda_{i,t}^0. \quad (\text{C.5})$$

The intertemporal factor in the Phillips equation is likewise unchanged. More generally, substituting the transformation T into the residual vector gives the identity

$$g_t^1(Ty) = D_t g_t^0(y), \quad d_{j,t} = \begin{cases} \mathcal{A}_t^{-1}, & j \in \{1, 5\}, \\ \mathcal{A}_t, & j \in \{8, 9, 13, 16\}, \\ 1, & j \in \{2, 3, 4, 6, 7, 10, 11, 12, 14, 15, 17\}, \end{cases} \quad (\text{C.6})$$

where D_t is diagonal with entries $d_{j,t}$ in the E1–E17 ordering. This identity holds off equilibrium on the positive domain. Since $\mathcal{A}_t > 0$, T is invertible and establishes a correspondence of private feasible paths. The domestic-price-stability restriction is preserved because π_h is unchanged.

Next consider the planner. For each household, $u_{i,t}^1(Ty) = u_{i,t}^0(y) + \log \mathcal{A}_t$. Therefore

$$U_t^1(Ty) = U_t^0(y) + a \log \mathcal{A}_t, \quad a = (1 - \lambda)\alpha_S + \lambda\alpha_B. \quad (\text{C.7})$$

The weights are the same in both economies and are normalized to $a = 1$. Indeed, changing γ_f does not change the zero-shock reference allocation used to construct the Negishi weights. The discounted objective differs by $a \sum_{t \geq 0} \beta_S^t \log \mathcal{A}_t$, an exogenous constant independent of allocations.

The transformation of planner multipliers is

$$\zeta_t^1 = D_t^{-\top} \zeta_t^0. \quad (\text{C.8})$$

It ensures $(\zeta_t^1)^\top g_t^1(Ty) = (\zeta_t^0)^\top g_t^0(y)$. The transformed Lagrangian therefore equals the original Lagrangian plus the exogenous utility term. Because the coordinate map has an invertible Jacobian and depends only on the given external path, all planner stationarity conditions correspond, including the instrument condition and the lagged and forward multiplier terms in (A.18).

Finally, transform the physical prehistory by T and the inherited multipliers by $D_{-1}^{-\top}$. The boundary contribution obeys

$$(\zeta_{-1}^1)^\top g_{-1}^1(Ty_{-2}, Ty_{-1}, Ty_0) = (\zeta_{-1}^0)^\top g_{-1}^0(y_{-2}, y_{-1}, y_0). \quad (\text{C.9})$$

In the paper's experiments, $x_{-2}^o = x_{-1}^o = 0$, so that $\mathcal{A}_{-2} = \mathcal{A}_{-1} = 1$. The physical prehistory and inherited multipliers are then exactly the same across economies. As the oil state returns to zero, $\mathcal{A}_t \rightarrow 1$, preserving the terminal reference state. The bijection of feasible paths, together with the allocation-independent difference in the augmented objective, gives the stated correspondence of optima whenever those optima exist. $\square$

The proposition explains why the additional import-cost scenario can lower composite consumption while leaving the corresponding Ramsey policy-rate and domestic-activity paths unchanged. It does not establish that a numerical stationary point is a global maximum. A CPI rule does not generally share the correspondence: the transformed inflation rate changes the rule's right-hand side. Nor does the result automatically extend to $\eta \neq 1$, $\sigma \neq 1$, imported production inputs, collateral valued at a fixed CPI-denominated price, or boundary conditions held fixed when they require transformation.

The scope of the foreign price also matters. If $v_{f,t}$ were the price of foreign competitors as well as Canadian imports, export demand would contain $p_{h,t}/(s_t v_{f,t})$. Exports would then change under (C.2), invalidating the feasible-set correspondence. The result is thus a separability property of this specific model and experiment, rather than evidence that imported inflation generally has no consequences for optimal monetary policy.

D Policy Comparisons and Numerical Implementation

D.1 Initial Conditions, Promises, and Consumption Equivalents

Let y_t collect physical endogenous variables and the exogenous states, and let $g(y_{t-1}, y_t, y_{t+1}) = 0$ denote the private equilibrium conditions in the sign convention used to construct the Ramsey Lagrangian. Every policy starts with the same stationary physical prehistory $\bar{y}^R$ and the same vector of inherited multipliers $\bar{\zeta}$. The criterion evaluated on policy p is

$$J_p = W_{p,0} + \beta_S^{-1} \bar{\zeta}' g(\bar{y}^R, \bar{y}^R, y_{p,0}). \quad (\text{D.1})$$

The second term records the effect of the date-zero allocation on inherited forward-looking commitments. It is retained for every candidate, including rules that do not themselves solve the Ramsey first-order conditions. This implements a common timeless comparison in the sense of Woodford (1999). Dropping that term would change the policy criterion.

Unaugmented social utility uses discount factor β_S and fixed Negishi weights normalized by $(1 - \lambda)\alpha_S + \lambda\alpha_B = 1$. Individual values V_i instead use the household's own β_i . With logarithmic consumption utility, the

exact reported percentage gain relative to policy q is

$$CE_i(p, q) = 100 \left\{ \exp[(1 - \beta_i)(V_{i,p} - V_{i,q})] - 1 \right\}. \quad (D.2)$$

For the social CE, replace V_i by W and β_i by β_S ; the weight normalization makes the same formula valid for a common proportional increase in both types' consumption. Housing and hours on the reference path remain fixed when constructing this compensating variation, although their utility changes enter the value difference. No CE is assigned to J .

Episode CEs compare a shocked path with the same policy's no-shock path from the common physical state. The no-shock paths generally differ across rules, because their terminal stationary equilibria differ. Accordingly, differences between episode CEs are not direct welfare gains from switching policies. For any value or level observable F , nonlinear shock interaction is computed as $F_{\tau,o} - F_{\tau,0} - F_{0,o} + F_{0,0}$. The baseline term is essential. Applying the exponential CE transformation to this utility interaction also differs from subtracting three episode CEs.

D.2 Stationary Branch and Equilibrium Solution

The Ramsey stationary calculation reduces the instrument condition to a scalar root search, conditional on the private stationary allocation and the associated multiplier system. Every candidate is checked against the full private and planner conditions, positive variable domains, and $\mu \geq 0$ before selection. A sign change across a nearly singular multiplier system is insufficient. The selected validated candidate is the one closest to the private reference rate. In the central calibration, two validated candidates are found, with quarterly gross rates approximately 1.0025581 and 1.0066107; the first is selected. This branch convention does not establish a globally optimal stationary allocation.

The reported episodes solve the full nonlinear deterministic boundary-value problem using sparse Newton updates, analytical derivatives, and each policy's stationary continuation at the terminal boundary. Continuation values are included in the utility recursions. The main horizon is 6,000 quarters; the main impulse-response figure displays the first 40 and the distribution figure the first 24. A converged equality solution is retained as an economic outcome only if consumption, housing, labor, prices, and gross rates have valid domains, the binding collateral equation holds, and the private multiplier remains nonnegative. No occasionally binding regime or effective lower bound is imposed. All 16 central paths pass these checks; their largest equation residual is 2.01×10^{-12} and their smallest μ is 0.00618.

The 16 paths comprise four policies and four scenarios, including no shock. The calibration study contains 54 paths: nine configurations, three policies, and shocked and no-shock paths. Its six central paths overlap the central comparison; these counts must not be interpreted as independent replications. The three common policies also agree across the main and rule-optimization pipelines to an absolute path difference below 1.1×10^{-13} .

D.3 Rule Search and Accuracy Checks

The rule search maximizes the joint-shock level of J over $\rho_R \in [0, 0.98]$ and $\phi_\pi \in [1.01, 8]$, subject to local determinacy and path admissibility. Screening and optimization use nonlinear paths of 2,000 quarters. Three Nelder–Mead starts converge to essentially the same interior solution. A Powell search approaches that value

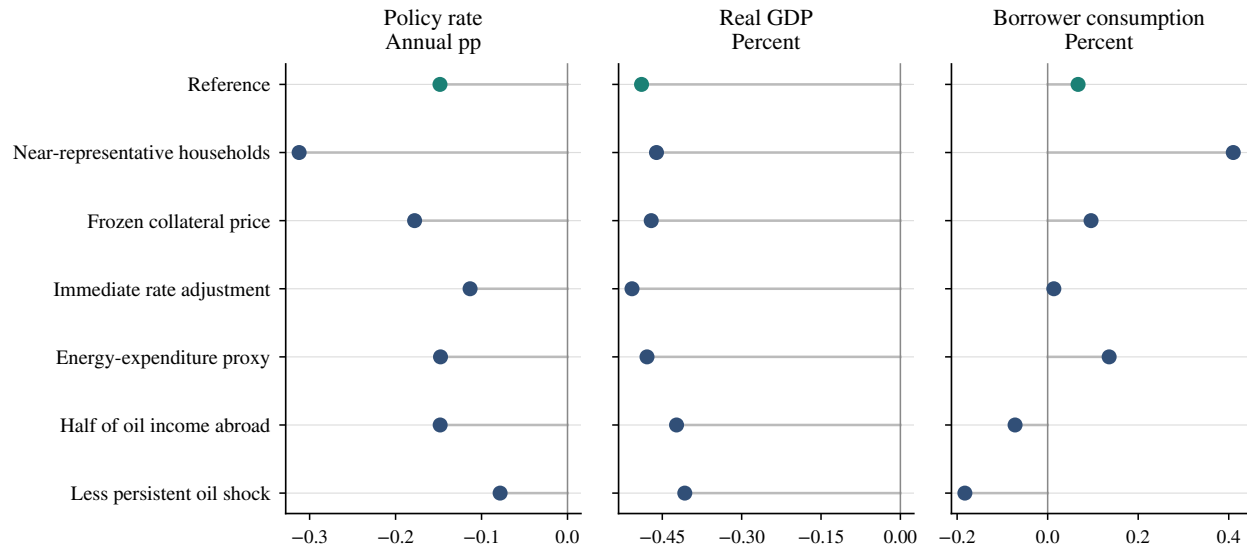


Figure D.1. Transmission Mechanisms and the Ramsey Impact Response

Notes: First-order responses to the joint shock, computed around each variant's own stationary allocation. The policy rate is in annual percentage points; real GDP and borrower consumption are in percent. The near-representative-household variant jointly reduces the borrower share and aggregate debt. Immediate rate adjustment sets $\rho = 1$ while retaining the reference amplitude. The less persistent oil shock has $\rho_o = 0.8$. The energy-expenditure proxy is a separate mechanism calibration from the supply-use alternative in Table 5. These local comparisons describe transmission within each variant; they are not cross-economy welfare rankings. Source: calibrated model.

but exhausts its evaluation budget. Eight small coefficient perturbations yield no improvement. There are 479 admissible objective evaluations, followed by the full 6,000-quarter comparisons; no second-order solution is rebuilt during this search. This is a bounded conditional search, not a global or stochastic optimal-rule calculation.

Increasing the final central comparison horizon from 4,000 to 6,000 changes J by at most 1.14×10^{-11} and the first 80 quarters of scaled physical paths by at most 1.32×10^{-10} . Ramsey has a slow stable mode, so a small terminal allocation gap remains even at the longer horizon. A separate 6,000-to-12,000-quarter check at a noncentral mortgage configuration with slow terminal convergence changes the Ramsey impact rate response by less than 1.6×10^{-11} annual percentage point and its social CE by less than 3.9×10^{-13} percentage point.

First- and second-order deterministic approximations provide additional accuracy checks: residual scaling exponents approach two and three, respectively, as shock sizes shrink. These approximations do not add future shock variance or precautionary welfare terms to the reported episodes. The archived suite passes 23 tests covering equilibrium aggregation, derivatives, contract timing, common-state welfare, solver failure guards, and root selection. Eight additional feasible perturbations of feedback around the Ramsey path lower J . They support the numerical comparison but do not constitute a second-order sufficient condition or a proof of global optimality.

Mechanism scope. The indexed-debt diagnostic is reported only under the CPI rule. The searched grid yields no validated utilitarian Ramsey root; removing inherited promises yields a negative borrowing multiplier. Both cases are excluded from the Ramsey comparisons, without implying equilibrium nonexistence.